ABSTRACTS OF PAPERS

to be presented at

the 51st Annual Conference of the

INDIAN MATHEMATICAL SOCIETY

to be held at Cochin

(28th, 29th and 30th December, 1985)

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(A) COMBINATORICS AND DISCRETE MATHEMATICS

1. **Some Congruences Via Combinatorics**—*Rajendran Valiaveetil* (*University of Calicut, Calicut*).

X denotes a finite set. $|X|$ represents the cardinality of X. Let p be a prime. Suppose $f: X \to X$ is such that $f^p = I$, the identity map. If

$$X_0 = \{x \in X \mid f(x) = x\}$$

it is known that (M. Hausner, Amer. Math. Monthly 1983)

(1) $$|X| \equiv |X_0| \pmod{p}.$$

For proof, see [1].

The purpose of this note is to illustrate the use of the above combinatorial principle in the following congruences :

(2) If $\phi(n)$ denotes Euler's totient, $\phi(n) \equiv 0 \pmod 2$ for $n \geqslant 3$.

(3) If $d(n)$ denotes the number of divisors of n, then

$$d(n) \equiv \begin{cases} 1 \pmod 2 & \text{if } n \text{ is a perfect square} \\ 0 \pmod 2 & \text{otherwise} \end{cases}$$

(4) $(p-1)!+1 \equiv 0 \pmod p$ (Wilson's Theorem)

2. **Strongly Indexable Graphs**—*B.D. Acharya and S. M. Hegde* (*M.R.I., Allahabad*).

A (p, q)-graph $G=(V, E)$ is said to be *strongly k-indexable* if it admits *strong k-indexer*, viz. an injective function $f: V \to \{0, 1, 2, \ldots, p-1\}$ such that $\{f(x)+f(y) : xy \in E\} = \{k, k+1, k+2, \ldots, k+q-1\}$. In the terms defined here, k is dropped if it is unity. We find that a strongly indexable graph is either a star or has a triangle, and if there is a triangle then there are at most $[(2/3)(p-3)]$ triangles in it, where $[\cdot]$ denotes the greatest integer function. Moreover, there exist infinitely many connected strongly indexable graphs G with $[2/3)(2|V(G)|-3)]$ triangles. In any strongly k-indexable graph minimum point degree is at most $3k$ and using this fact we show that there

are exactly three strongly indexable regular graphs viz., K_2, K_3 and $K_2 \times K_3$. If an eulerian (p, q)-graph is strongly indexable, then $q \equiv 0, 3 \pmod 4$. Lastly, we give a matrix method to generate all strongly indexable graphs on a given number of points.

3. Independent, Perfect and Connected Neighbourhood Numbers of a Graph *Prabha S. Neeralagi (Karnatak University, Dharwad).*

A set S of points in a graph G is an *n-set* if $G = \bigcup_{v \in S} <N(v)>$, where $<N(v)>$ is the subgraph of G induced by v and all points adjacent to v. The *neighbourhood number* $n_0(G)$ of G is the minimum cardinality of an n-set. *The independent neighbourhood number* $n_{oi}(G)$ of G is the minimum cardinality of an independent n-set. The *perfect neighbourhood number* $n_{op}(G)$ of G is the minimum cardinality of an n-set S such that for all $u, v \in S$, $u \neq v$, $<N(u)>$ and $<N(v)>$ are line disjoint. *The connected neighbourhood number* n_{oc} G) of G is the minimum cardinality of an n-set S such that the subgraph S of G induced by S is connected. The number $n_0(G)$ is studied in [4]. Here the numbers $n_{oi}(G)$, $n_{op}(G)$ and $n_{oc}(G)$ are determined for some known graphs and some bounds are obtained for them. Also, Nordhaus-Gaddum type results are established for these numbers.

4. Forbidden Subgraphs for Minimally Non-Outerplanar Middle Graphs—*V. R. Kulli (Gulbarga University, Gulbarga) and H P. Patil (P. G. Centre, Gulbarga University, Nandihalli Sandur).*

In this paper, we establish a characterisation of graphs with minimally non-outerplanar middle graphs in terms of forbidden subgraphs.

5. The Line Neighbourhood Number of a Graph—*E. Sampathkumar and Prabha S. Neeralagi (Karnatak University, Dharwad).*

For a point u and a line $x = uv$ in a graph $G = (V, E)$, let $N(u) = \{v \in V : uv \in E\}$ and $N[x] = N(u) \cup N(v)$. A set $T \subset E$ is a *line-neighbourhood set* (written *ln-set*) of G if $G = \bigcup_{x \in T} <N[x]>$, where $<N[x]>$ is the subgraph of G induced by $N[x]$. The *line-neighbourhood number* $n_0'(G)$ of G is the minimum cardinality of a *ln*-set. Besides investigating some relationship of $n_0'(G)$ with other known parameters of G, many bounds for $n_0'(G)$ are obtained.

6. A Note on Unitary Divisors and Bi-Unitary Divisors—*V. Bhaskara Reddy and S. Vangipuram (S.V. University College, Tirupati).*

A positive integer d is called a unitary divisor of n if d is a divisor of n such that $\left(d, \frac{n}{d}\right) = 1$ and this is denoted by $d \parallel n$. Chidambara swamy [2] has defined a divisor $d > 0$ of the positive integer n to be a semi-unitary divisor of n if $\left(d, \frac{n}{d}\right)^* = 1$, where $(a, b)^*$ is defined to be the greatest divisor of a which is a unitary divisior of b. If $(a, b)^* = 1$, then a is said to be semi-prime to b. Analogous to the Euler-totient function $\phi(n)$, Cohen [1] has defined $\phi^*(n)$to be the number of numbers $\leqslant n$ and which are semi-prime to n. He has proved that

$$\phi^*(n)=\sum_{d/n} d\mu^* \left(\frac{n}{d}\right),$$

where $\mu^*(n)=(-1)\, w(n)$ where $w(n)$ is the number of distinct prime factors of n. In this note we have obtained a relationship between $\phi^*(n)$ and $\phi(n)$, an expression for $\phi^*(n)$, and some necessary conditions for a number to be a unitary perfect totient number.

7. **A Note on a Theorem of Perron**—*K.C. Prasad and M. Lari (Ranchi University, Ranchi).*

Given an infinite simple continued fraction $[a_0, a_1, \ldots, a_n, \ldots]$, let M_n denote $[0, a_n, a_{n-1}, \ldots a_1]+[a_{n+1}, a_{n+2}, \ldots]$. A well known result due to perron states : If $a_{n+2}=m$ then there is a k in $\{n, n+1, n+2\}$ for which $M_k>\sqrt{m^2+4}$. In this note we give a new proof for this result and add that : there is a j in $\{n, n+1, n+2\}$ for which $M_j<\sqrt{m^2+4}$.

8. **Two families of simple and irreducible BIBDs**—*V.N. Bhat-Nayak and V D. Kane (University of Bombay, Bombay).*

If $v\equiv 1$ or 3 mod 6, then $(v, 3, 2)$-design is a quasi -2-multiple design. We give here a direct construction of simple and irreducible $(v, 3, 2)$-designs when $v\equiv 1, 3$ mod 6 and $v\neq 7$.

(B) ALGEBRA, NUMBER THEORY, AND LATTICE THEORY

9. **Palindromic Numbers by Demlofication**—*D. R. Kaprekar (Abhinav Bharat Mandir, Nashik City).*

Let us consider a number of two digits AB. A is in Tens place and B is in Units place, and such that $A+B=11$. (The digit Zero is not allowed.)

Let us take 56. Here $5+6=11$ and $B=6$.

From 56 let us prepare 6 numbers in Arithmetico Geometrical process. These will be called 6 terms.

1st term $=56$

2nd term $=(56+99)\times 10^1=1550.$

3rd term $=(56+2\times 99)\times 10^2=25400.$

4th term $=(56+3\times 99)\times 10^3=353000.$

5th term $=(56+4\times 99)\times 10^4=4520000.$

6th term $=(56+5\times 99)\times 10^5=55100000.$

Now leftward demlofication of these 6 terms means writing these terms diagonally one below above but shifting each time one place towards left ; it will be seen as

$$\begin{array}{r} 56 \\ 155 \\ 254 \\ 353 \\ 452 \\ 551 \\ \hline 60000006 \end{array} = 6(0)_6\ 6 = B(0)_B\ B$$

(Where $(0)_B$ means 0 is repeated B times.) Here 6 is the value of B.

If AB is such as $A+B=12$ or 13, 14, .. , or $11+K$ $(K<B)$, the process will give $B\,(A+B-11)_B\ B$.

10. Fuzzy Prime Ideals of Rings—*U.M. Swamy and K.L.N. Swamy (Andhra University, Waltair).*

The relation between prime ideals and fuzzy prime ideals of rings is studied. It is shown that if P is a fuzzy ideal of a ring R, then, P is fuzzy prime iff there exists a prime ideal Q of R and α in [0, 1] such that $P(x)=1$ if x is in Q and α if x is not in Q. The non-existence of maximal fuzzy ideals is also established with the natural meaning of a fuzzy maximal ideal as a maximal element among fuzzy ideals.

11. Fuzzy Equivalences and Fuzzy Congruences—*K.L.N. Swamy and U.M. Swamy (Andhra University, Waltair).*

The relation between ordinary equivalence relations and fuzzy equivalences is studied. The notion of a fuzzy congruence on a Ω-algebra is introduced and it is shown that they are built up by certain chains of ordinary congruence relations on the given Ω-algebra. Specifically if $\triangle$ is a nonempty nonzero subset of [0, 1] and $\{\theta_\alpha/\alpha \in \triangle\}$ is a family of congruences on a Ω-algebra M such that (i) $\alpha \leqslant \beta \Rightarrow \theta_\beta \leqslant \theta_\alpha$ (ii) for each $(x, y) \in M \times M$, there exists largest α in $\triangle$ such that $(x, y) \in \theta_\alpha$, then the maping $\theta : M \times M \to [0, 1]$ defined by $\theta(x, y)$=the largest α in $\triangle$ such that $(x, y) \in \theta_\alpha$ is a fuzzy congruence on M and conversely any fuzzy congruence can be constructed as above.

12. Lattice Structure to Material Universe—*P.K. Lal and S.P. Bhagat (Bhagalpur University, Bhagalpur).*

The bodies which mechanics deals are of different types : such as masspoints, rigid bodies, strings, rods, jets, shells, space filling fluids, solids etc. We denote the bodies by the symbols A, B, C, .. The paper proposes to assign the structure of Boolean lattice (or complemented distributive lattice) to the bodies in general.

13. Structure of Universal Derivation Module—*H.V. Kumbhojkar (Shivaji University, Kolhapur).*

Let a diagram $< A_\alpha, \phi_{\beta\alpha} >$ over a diagram scheme $\Sigma=(I, M, D)$ in the category of commutative unitary algebras have a colomit algebra A. It is then proved that the universal derivation module $u\,(A)$ over A is a colomit of a diagram over Σ in the category of derivation modules such that for each $\alpha \in I$ $u\,(A_\alpha)$ is a universal derivation module over A_α. This is achieved by showing that the functor from the category of derivation modules into the category of algebras admits a split normalized cleavage.

14. Noetherian Intermediate Rings—*D. Alamelu* (*Ramanujan Institute, Madras*).

We call an extension ring S of R an NA extension if every noetherian subring of S containing R is affine over R. If N is a m c set in R, then S_N is an NA extension of R_N. Conversely if S is affine over R, and S_N is an NA extension of R_P for each maximal ideal P of R, then S is an NA extension of R. Every 1 dimensional affine ring over a field k is an NA extension over k. An example given by Wadsworth shows that there exist noetherian subrings which are not affine inside $K[x, y]$ where k is sufficiently large.

15. On Some Isotopies of Planar Ternary Rings With Zero of A Moufang Plane—*M. Bommi Reddy and L. Nagamuni Reddy* (*S. P. Mahila Visva-Vidyalaya, Tirupati*).

The algebraic properties of the coordinatizing Planar Ternary Rings with zero (PTRZ's) of translation, Desarguesian and Pappian planes are studied by Klucky and Markova (Czech. Math. J. 23 (98), 1973, 24 (99), 1974) and that of Moufang planes by M. Bommireddy, L. Nagamuni Reddy and K. Sitaram (Arch Math. 1985). Klucky established the equivalence of the corresponding properties of PTRZ's and Hall Ternary Rings of translation, Desarguesian and Pappian planes by means of isotopies. In this paper a similar study is done for Moufang planes.

16. Statistisch Pairs, p-Compatible Pairs in Atomistic Lattices and some Generalizations—*N K. Thakare* (*Univesity of Poona, Pune*) *and M. P. Wasadikar* (*Aurangabad*).

This is continuation of the study of the theory of symmetricity in join semilattices carried out in Thakare, Wasadikar and Maeda [Algebra Universalis **19**, (1984)], and Meada, Thakare and Wasadikar [Algebra Universalis, **20** (1985)] on p-compatible AC-join semilattices, finite statisch—AC join semilattices. Strongly p-compatible semilattices are further investigated and it is proved among other results that an element c in an AC join semilattice L is an atom of (a) if and only if there exists an atom p in L such that $c = aVp$ and $p \leqslant a$. Several novel notions such as p-elements, statisch pairs, finite statisch pairs, and p-compatible pairs etc. are introduced and are studied from many angles

17. A Sufficient Condition for the Uniqueness of the Base of a Unique Representation set—*S. Audinarayana Moorthy* (*Kendrapara College Kendrapara*).

In this paper we define a *unique representation set* (u. r. set), a (simple linear) *unique representation base* or simply a *base*, and (simple linear) *unique representation set, with base B*. A u. r. set need not have a unique base. The following theorem gives a fairly general sufficient condition for the uniqueness of the base. *Theorem.* Let (G, +) be an abelian group with identity, 0, and P, a subset of G, containing 0, closed w.r.t. '+', and linearly ordered w.r.t. a relation "$\leqslant$", such that, for every a, b, c $\in P$, (1) $0 \leqslant' a$ and (2) $a + b \leqslant' a + c$ implies $b \leqslant' c$. Also, let S, a subset of P, be a u r. set w.r.t. '+', whose elements can be listed according to the order relation $\leqslant'$. Then S has a unique base.

18. Equivalence of M-Symmetry and Semimodularity in Lattices—*Chinthayamma Malliah and S. Parameshwara Bhatta* (*Mangalore University, Mangalore*).

G. Gratzer had posed the following problem : Is M-symmetry equivalent to semi-modularity for algebraic lattices ? This paper gives a negative solution to this problem by constructing a counter example. Further the equivalence of M-symmetry and semi-modularity in the case of weakly atomic continuous lattices is established. As a consequence certain sufficent conditions for algebraic semi-modular lattices to be M-symmetric are deduced.

19. A Condition for the Commutativity of Rings—*Murtaza A. Quadri and Mohd. Ashraf (Aligarh Muslim University, Aligarh).*

Recently the authors [Bull. Austral. Math. Soc. 31 (1985), 365—368] improved a theorem of Abu-Khuzam and Yaqub [Bull. Austral. Math Soc. 21 (1980), 43—46) which in turn generalizes a well-known result due to Israel N. Herstein [Atti. Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) 32 (1962), 177—180] that a division ring D in which for each x, y in D, $xy-yx$ is central must be commutative. Infact, we proved : "If R is a semi-simple ring in which for each x, y in R there exists a positive integer $n=n(x, y) > 1$ for which either $[z, (xy)^n + (yx)^n] = 0$ or $[z, (xy)^n - (yx)^n] = 0$ for all z in R, where $[x, y] = xy - yx$, then R is commutative." Our present objective is to generalize the above theorem as follows : THEOREM : Let l, m, n be fixed positive integers and R be a semi-prime ring in which $[(xy)^l, (xy)^m - (yx)^n] = 0$, for all x, y in R, then R is commutative.

20. A Commutativity Theorem for Semi Prime Rings—*Murtaza A. Quadri and Moharram A. Khan (Aligarh Muslim University, Aligarh).*

Let R be an associative ring and for $x, y, \in R$, $[x, y]=yx-yx$. We prove the following theorem which extends many known results particularly the theorem of Herstein [Canad. J. Math. 7 (1955), pp. 411—412]. THEOREM : In a semi prime ring R, the following conditions are equivalent : (1) R is commutative. (2) For each x, y in R, there exists a positive integer $n > 1$ such that $[(xy)^n - xy, x] = 0$ or $[(xy)^n - xy, y] = 0$. (3) For each x, y in R, there exists a positive integer $n > 1$ such that $[(xy)^n - yx, x] = 0$ or $[(xy)^n - yx, y] = 0$. (4) There exists a positive integer $n > 1$ such that $[(xy)^n - yx, xy] = 0$ for all x, y in R.

21. Fuzzy Vector Spaces Under Triangular Norms—*Phullendu Das (Visva-Bharati, Santiniketan).*

Using the notion of a triangular norm, fuzzy subspaces are redefined and some generalisations of the results of Katsaras and Liu (Fuzzy vector spaces and fuzzy topological vector spaces, J. Math. Anal Appl. 58 (1977), 135–146] are obtained.

22. Fuzziness in Forest Language—*Rashmi Lal and R. N. Lal, (Bhagalpur University, Bhagalpur).*

The notion of tree and forest language has been introduced by Brainerd, W. S. (1968). Fuzziness in a language leads to fuzzify a forest language. The paper is concerned with an introduction of a fuzzy forest language.

23. **Fuzzy NHD Lattice**—*P. K. Lal and R. N. Lal (Bhagalpur University, Bhagalpur).*

Our notion of fuzziness in a lattice leads to fuzzify our nhd lattice. A fuzzy nhd structure on an atomistic lattice is mapping on the set of fuzzy atoms into subsets of fuzzy elements. The paper investigates the properties of this new structure.

24 **n-Metric Lattice Associated with (n+1)—ary Fuzzy Dissimilarity**—*D. N. Singh and R.N. Lal, (Bhagalpur University, Bhagalpur).*

Associated with our $(n-1)$ ary atomic fuzzy dissimilarity, a n—metric d can be introduced on an atomistic lattice p. We study in this paper, the properties of n-metric lattices mentioned in the title of the paper.

25. **Fuzzy Proximal Lattice**—*P.K. Sinha and R.N. Lal (Bhagalpur University, Bhagalpur).*

Our introduction of fuzziness in a lattice leads to the fuzzification of our proximal lattice which is the object of our study in this paper.

26. **Fuzzy Uniform Lattice**— *P.K. Sinha and R.N. Lal (Bhagalpur Univesity, Bhagalpur).*

Our introduction of fuzziness in a lattice leads to fuzzify our uniform *nhd* lattice. In this paper we study fuzzy uniform lattices.

27. **Fuzzy Non-Standard Modelling of Axiomatic Set Theory**—*P.K. Sinha and R. N. Lal (Bhagalpur University, Bhagalpur).*

Fuzzy sets of higher level with fuzzy identity $\underset{\sim}{=}$ and fuzzy membership relation $\underset{\sim}{\in}$ are useful in modelling weak logic *WL* and Axiomatic Set Theory. Let F be universe of fuzzy sets of higher level defined by transfinite recursion. The paper is concerned with an introduction of fuzzy set structure $<F, \underset{\sim}{\in}, \underset{\sim}{=}>$ which will model nonstandard axiomatic set theory as well as weak logic.

28. **Fuzziness in Neighbo-Form Lattice**—*P.K. Sinha and R. N. Lal (Bhagalpur University, Bhagalpur).*

Our neighboformity (Cal. Math Soc. 1977) is a generalisation of Docinov generalised topology (Sov. Math, 19.4). Our fuzziness in a atomistic lattice leads to the fuzzification of such a neighboform lattice. Here we investigate its properties.

29. **Fuzziness in Probability Algebra**—*P.K. Sinha and R. N. Lal (Bhagalpur University, Bhagalpur).*

A probability space can be represented by a probability algebra which is a Boolean lattice with a strictly positive, normed and additive real valued function. A probability space has been fuzzified by Zhende Huang (198.). The paper is concerned with an introduction of fuzzy probability algebra.

30. On a Problem of B. Zelinka-I—*V.R. Chandran* (*University of Madras, Madras*).

In a paper by B. Zelinka, 'Tolerances in algebraic structure'—Czech. Math. journal (1975) 175—178, he has posed an open problem viz. whether $\exists$ a commutative semigroup in which each tolerance relation on its element set is compatible with it ? We solve this problem affirmatively by producing an example of such a semigroup.

31. On a Problem of B. Zelinka II—*V. R. Chandran* (*University of Madras, Madras*).

The purpose of the present paper is to give a complete characterisation of B. Zelinka's problem. THEOREM : Let $< S, * >$ be a commutative semigroup with a multiplicatively zero element Let $| S | \geqslant 3$. Then every tolerance relation in $<S, *>$ is compatible with its element set if and only if the product of any two elements in $< S, * >$ is zero.

32. On betweenness and Partial Orderings in Lattices—*P.V. Ramana Murty and Sr. Teresa Engelbert* (*Andhra Universiiy, Waltair*).

In his paper 'Betweenness isomorphism of modular lattices' [Archiv Der Mathematik, Vol. 37; 154—162 (1981)] J. Hedlikova has introduced a partial ordering in terms of lattice betweenness in modular lattices. In this paper we study this partial ordering in general lattices. Also characterizations of standard, dually standard and neutral elements are obtained in terms of lattice betweenness. Finally a characterization of modular lattices is given in terms of betweenness isomorphism and isomorphism of partial orderings.

33. M. Faithful Modules—*R.S Singh* (*University of Saugar, Sagar*).

In this paper annihilators are studied relative to tensor product. Among other results it is proved that if M is a flat R-module over Pruffer domain R, then any finitely presented module U is M-faithful if and only if U is faithful relative to all M-duals.

34. A Note on 'Lying Over Pairs'—*S. Visweswaran* (*Saurashtra Univ., Rajkot*).

Let R be a commutative ring with identity. Let S be a subring of R containing the identity of R. Recall from (1) that the pair (S, R) is a 'lying over pair' (*LO* pair) if for any pair of rings S_1, S_2 such that $S \subseteq S_1 \subseteq S_2 \subseteq R$ and any prime ideal P of S_1 there exists a prime ideal Q of S_2 such that $Q \cap S_1 = P$. Dobbs (Canad J. Math. 1981) proved that if (S, R) is an *LO* pair, then Krull dimension of $S \leqslant$ Krull dimension of $R \leqslant$ (Krull dimension of S)$+1$. In this note we prove for an *LO* pair (S, R), if either Krull dimension of $S \geqslant 1$ or if Krull dimension of $R > 1$, then Krull dimension of S=Krull dimension of R. Further we provide a negative answer to a question posed by Dobbs.

35. More Genuine First Order Models of Mathematical Studies, Appropriate Semantics, Rich Non-Standard Models and User Friendly Aspects—*J. Gopala Krishna and M. Usha Sree Devi* (*Andhra University, Waltair*).

It is observed that the significant mathematical study expressible in terms of first order structures is more naturally handled through generalized first

order structures which are the triples $\mathbf{M}=(M, O, R)$, where M is a nonempty set, O is a set of finitary partial operations on M(that is, M-valued mappings with domains in finite cartesian products of M with itself, so that the division may be accommodated in O when M is the set of real numbers) and R is a set of finitary relations on M. It is emphasized that the appropriate semantics should accommodate "undefined terms" and "meaningless sentences", in particular. Rich non-standard models $^*\mathbf{M}=(^*M, \hat{O}, \hat{R})$ for such a generalized structure $\mathbf{M}$ are considered in term of ultrapowers and direct limits using the ideas exposed by E. Nelson (Internal Set Theory : A new approach to non-standard analysis, Bulletin Amer. Math. Soc., Vol. 83, No. 6, 1977, pp. 1165-1198) where it may be particularly observed that the concurrence/idealization principle need not "cover" the entire M. When the relation of membership restricted to M^2, namely, $\epsilon(M)=\{(x, y) \in M^2 : x \in y\}$ is in R, the feasibility of constructing a non-standard $^*\mathbf{M}$ which would accommodate $\epsilon(^*M)$, as the associate of $\epsilon(M)$ and which retains as much generality as possible of the concurrence/idealization principle, is discussed using the ideas vividly exposed by Martin Davis.

36. **A Generalization of Frobenius Reciprocity Theorem**—*I.V.V. Raghavacharyulu* (*Bhabha Atomic Research Centre, Bombay*).

Frobinius reciprocity theorm states that $(\triangle_H{}^G, \triangle_G)=(\triangle_H, \triangle_G H)$. It may be looked upon as a recipe to evalute $(\triangle_H{}^G, \triangle_G)$ in terms of the representation $\triangle_H$ and the subduced representation $\triangle_G H$. A generalization of the Frobinius reciprocity theorem is obtained by establishing a recipe to evaluate $(\triangle_{H'}{}^{G}, \triangle_{G'}{}^{G})$ in terms of the represeatation $\triangle_{H'}$ and the direct sum of subduced representations $\sum_D \triangle_{G'}(D)$, where $\{D\}$ is the set of all $(H', G')^-$ double cosets of G. Hence the generalized Frobinius theorem states that

$$[\triangle_{H'}{}^{G}, \triangle_{G'}{}^{G})=\sum_{\{D\}}(\triangle_{H'}, \triangle_{G'}(D)],$$

where $\{D\}$ is set of all $(G', H')-$double cosets of G. This generalization is useful to establish the compatibility relations between a complete set of non-equivalent allowable irreducible representations of a group of second kind G^{II} in terms of a complete set of non-equivalent allowable irreducible representations of a subgroup H^{II} of G^{II} containing a normal subgroup N of G when G is reduced with respect to the normal subgroup N of G. When $N=\{e\}$, the generalization obviously corresponds to taking $G''=G$ and hence reduces to the usual Frobinius reciprocity theorem. The generalization has applications in physical contexts.

37. **On Associativity of Isotopes of Certain Near-Rings**—*T. Tamizh Chelvam* (*Annamalai University, Annamalai Nagar*).

The object of this paper is to study about isotopes of certain type of Near-rings N. First we obtain a necessary and sufficient condition for a general isotope of an associative near-ring with identity to be associative. Next we consider a particular type of near-ring N and prove two theorems, of which the first one gives all the possible associative special isotopes of N and the second one provides the necessary and sufficient condition for two special isotopes to be isomorphic.

38. **Embedding of a Near-Ring in a Loop Near-Ring**—*G. Koteswara Rao and K. Rajasekhar (Nagarjuna University, Nagarjuna Nagar).*

It is shown that an arbitrary non zero near-ring can be embedded in a loop near-ring which is not a near-ring

39. **Morita Equivalence for a Larger Class of Rings and Picard Group**—*M. Parvathi and A. Rama Krishna Rao (Ramanujan Institute, Madras).*

It was proved by the authors (pre print) that gamma context equivalence between rings, not necessarily containing identity but satisfying the gamma conditions ($R^2=R$ and $rR=0$ or $Rr=0 \Rightarrow r=0, r \in R$), implies Morita equivalence. In this paper we prove the converse. That is, Morita equivalence between rings implies gamma context equivalence. We also construct the Picard group of a ring satisfying the gamma conditions and prove that the Picard groups of Morita equivalent rings are isomorphic.

40. **A Note on Invariants of Rings**—*J. H. Pathak (University of Bombay, Bombay).*

Let A and B be two reduced noetherian subrings of a commutative ring R with infinitely many elements in common. Suppose

$$A[X_1 \ldots X_n] = B[Y_1 \ldots Y_n]$$

where X_i and Y_i are polynomial variables over A and B respectively, $i=1, 2, \ldots n$. Then there exists an isomorphism

$$\phi : A \to B$$

such that localization B_g is integral over $\phi(A)$ for some non-zero divsisor g in A.

41. **An Alternating Sum Formula in Differential Calculus**—*D.N. Verma (T I F.R., Bombay).*

By the Gelfand-Grothendieck philosophy, any commutative C-algebra is eligible to be called, and treated as, the ring of functions on a suitable 'space' (its "affine spectrum" over C), and more generally C can be replaced by any (commutative) base ring k ; however, for purpose of what follows we need not enter deeper into this basic scheme-theoretic point of view.

Let A be a commutative k-algebra (such as the polynomial algebra $k[X_1, X_2, \ldots, X_n]$). It is standard, and readily checked, that a k-endomorphism d of A is a first-order 'differential operator' (D. O. in short) $\Leftrightarrow$ $d(\varphi \cdot \psi) = (d\varphi)\cdot\psi + \varphi\cdot(d\psi) - \varphi\cdot\psi\cdot d(1_A)$ for all $\varphi, \psi \in A$; and that is so $\Leftrightarrow$ $d(\varphi_0\cdot\varphi \cdot \ldots \varphi_m) = \sum_{i=0}^{m} \varphi_0 \ldots \hat{\varphi}_i, \ldots \varphi_m\, d(\varphi_i) - m\varphi_0\cdot\varphi_1 \ldots \varphi_m\, d(1_A)$ for $\forall m \geqslant 1$ with $\varphi_0, \varphi_1, \ldots \varphi_m \in A$. Here we prove that an endomorphism D is a q^{th} order D.O $\Leftrightarrow$ for all $\varphi_0, \varphi_1, \ldots, \varphi_m \in A$ with $m \geqslant q$ one has

$$D(\varphi_I) = \sum_{j=0}^{r} (-)^{r-j} \binom{m-j}{r-j} \cdot \sum_{S \in \binom{I}{j}} \varphi_S \cdot D(\varphi_S),$$

$\Leftrightarrow$ the same holds for $m=q$; here $I=\{0, 1, \ldots, m\}$ the notation $\binom{I}{j}$ stands for all size j subsets of I, φ_S is an abbreviation for $\varphi_{i_1}\cdot\varphi_{i_2}\cdot \ldots, \varphi_{i_j}$ in case

$S=\{i_1, i_2, \ldots, i_j\} \in \binom{I}{j}$, while S' is the complement of S in I, and finally $\binom{a}{b}$ is the usual binomial coefficient.

For arbitrary A as above, the general notion of q^{th} order D.O. is obtained recursively as an endomorphism D on A such that the Lie-bracket $D\circ\mu-\mu\circ D$ is a $(q-1)^{th}$ order D.O. for arbitrary multiplication operator $\mu=\mu_\varphi : A\to A$ sending $\psi\to\varphi\psi$, with the understanding that the zero endomorphism is the only D.O of order -1. However, the result obtained is significant even for the polynomial algebra case. Even in full generality, our theorem belongs to the genre of "High School mathematics".

42. Localization of Skew Group Ring—*M. Mary John and M. Parvathi (Ramanujan Institute, Madras).*

We take G to be a finite group of automorphisms acting on a non-commutative ring R with unity and $R*G$ the skew-group ring of G over R. We introduce the concepts of G-invariant and G-prime torsion theories and get a relationship between the G-invariant torsion theories of R and the torsion theories of $R*G$. When τ is a G-invariant torsion theory of R we prove that the action of G extends to R_τ and that $R_\tau*G\simeq(R*G)\gamma_{\#}(\tau)$.

We also prove that if R is seminoetherian, then $R*G$ is seminoetherian. Using this and following D.S. Passman we are able to prove that when R is seminoetherian, for any G-prime torsion theory τ of R there exists a prime torsion theory σ of $R*G$ such that $\gamma^{\#}(\sigma)=\tau$.

43. On Isotopes of Arbitrary Near-Rings—*N. Ganesan (Annamalai Usiversity Annamalai Nagar).*

The object of this paper is to study about the isotopes of arbitrary near-rings. This notion helps us to construct arbitrary near-rings from associative near-rings. In this paper we prove that any special isotope of a cyclic ring is associative and establish theorems concerning general isotopes of N. In particular we give a necessary and sufficient condition for two general isotopes of N to be isomorphic in terms of the automorphism of $(N, +)$.

44. The Quotient Formula for the Schur Complement in an EP Martrix—*A. R. Meenakshi (Annamalai University, Annamalai Nagar).*

Let M be a complex matrix partitioned into the form $M=\begin{bmatrix}A & B\\ C & D\end{bmatrix}$. Then the Schur Complement of A in M is $M/A=D-CA^+B$, where A^+ is the Moore Penrose inverse of A. A square matrix is called EP if A and its conjugate transpose A^* has the same null space. Suppose A is of the form $A=\begin{bmatrix}B & G\\ H & K\end{bmatrix}$, where $B=\begin{bmatrix}C & D\\ E & F\end{bmatrix}$. It is shown that under certain conditions B, $C.B/C$, A/C and $A/B=(A/C)/(B/C)$ are all EP whenever A is EP. Some particular cases of this result are studied.

45. Positive Values of Inhomogeneous Quadratic Forms of Signature–2—*Satish K. Agarwal and D. P. Gupta* (*M. D. University, Rohtak*).

Let $Q(x_1, .., x_n)$ be a real indefinite quadratic form of type $(r, n-r)$ $(1 \leqslant r \leqslant n-1)$ and $D \neq 0$. Then we know there exist constants Γ independent of Q and depending only on n and r such that given any reals $C_1, .., C_n$; there exist $(x_1, ..., x_n) \equiv (C_1, ..., C_n) \pmod 1$ satisfying

$$0 < Q(x_1, ..., x_n) \leqslant (\Gamma \mid D \mid) \frac{1}{n}.$$

Let $\Gamma(n-r)$ denote the infimum of all such numbers Γ. Then we know the values of Γs, r $(s = r-1, r+1, r+2, r+3)$. We prove the following results. Theorem 1 : $\Gamma 4$, $6 \leqslant 2^9$. Theorem : Γr, $r+2 = \frac{2^{2r+2}}{3}$ $(r \geqslant 5)$ and the result is best possible. Thus we have proved the conjecture of Bambah, Dumir and Hans-Gill for $n \geqslant 12$.

46. N-Groups with Finite Goldie Dimension—*Venkateswara Reddy Yenumula and Satyanaryana Bhavanari* (*Nagarjuna University, Nagarjuna Nagar*).

The aim of this paper is to prove :

THEOREM : Suppose G is an N-group (i e., a near-ring-module) in which every essential N-subgroup is strictly essential. Then the following conditions are equivalent :

(*a*) G has Finite Goldie Dimension.

(*b*) G contains a finite number of uniform ideals whose sum is direct and essential in G.

(*c*) G contains a finite number of strictly uniform ideals whose sum is direct and essential in G.

47. Some Properties of Reflective Subcategories—*S. M. A. Zaidi* (*Aligarh Muslim University, Aligarh*).

The concept of reflections, reflective subcategory and their duals was introduced by P. J. Freyd in his Ph D. thesis in 1960. In this note we have shown the existence of reflection by generalized pushout, and obtained several properties of reflective subcategories.

The main results of this note are as follows :

THEOREM 1. Every *GPO*-dense subcategory which is also a *GPO*-category is reflective. Conversely, every reflective subcategory of a *GPO*-category is also a *GPO*-category.

THEOREM 2. Every full reflective subcategory of a *GPB*-category is also a *GPB*-category.

THEOREM 3. Every reflective subcategory of a right complete category is also right complete.

48. **Some Generalizations in Abelian Groups**—*M. Zubair Khan* (*Aligarh Muslim University, Aligarh*).

S. Singh in Proc. of Ohio University N.Y. (1976) studied two conditions on a module M_R and the author proved some decomposition theorems in Can. Math. Bull. (1979) and Math. Japonica (1978, 1979) for these modules. One of the main purposes of this paper is to generalize a concept introduced by K. Honda. The main result proved in this paper is the following characterization :

THEOREM : If N is a submodule of M, then following are equivalent :

(*a*) N is h-pure in M.

(*b*) N is a direct summand of $H^n(N)$ for every $n>0$.

(*c*) If $N \subseteq K \subseteq M$ and K/N is jointly generated, then N is a direct summand of K.

49. **Antipower Maps and Commutativity of Rings**—*Murtaza A. Quadri and Mohd. Ashraf* (*Aligarh Muslim University, Aligarh*).

A ring R is said to be (n, k)-ring, if for all $x, y \in R$, $(xy)^m = x^m y^m$, for the integers m with $n \leqslant m \leqslant n+k-1$, and dually (n, k)-ring if $(xy)^m = y^m x^m$. Several authors (Herstein, Bell, Luh, Awtar, Hermanci and Hazar Abu-Khuzam) have considered the commutativity of (n, k)-rings for $k = 1, 2, 3$. Recently the authors proved that a semi prime ring with unity satisfying $(xy)^2 = y^2x^2$, for all x, y in R must be commutative. In the present paper we prove the following Theorems :

THEOREM 1. Let R be an associative ring with unity. If R is also $(n, 3)^*$-ring, then R is necessarily commutative.

THEOREM 2 Let R be an associative ring with unity. If R is also $(n, 2)^*$-ring satisfying the condition $(x+y)^n = x^n + y^n$, then R is commutative.

THEOREM 3. Let R be a semi-prime ring (may be without unity). If R is also $(n, 1)^*$-ring, then R is commutative.

50. **On H^t_p-Groups**—*Asif Mashood* (*Aligarh Muslim University, Aligarh*).

In this paper groups all of whose pure-high extensions by torsion groups are splitting have been discussed. Such groups are named H^t_p-groups. Homological methods and technique are used throughout this paper. It has been proved that a group G is a H^t_p-group if and only if $Hext_p\{Z(p^\infty), G\}=0$. Elementary properties of H^t_p-groups are recorded. A torsion-free group is H^t_p-group. The role played by H^t_p-groups in algebraic compactness, group of extensions and pure extensions are studied.

(C) REAL ANALYSIS AND COMPLEX ANALYSIS

51. On the Integral Transforms of the Product of Generalized Functions—*T. Mohamed* (*Sir Syed College, Taliparamba*).

The object of this paper is to evaluate certain integral transforms involving the product of Fox's H-function and generalized functions of one and two variables of different arguments.

$$\text{(i)}\quad L\left\{ t^{\lambda-1} H_{P,Q}^{M,O}\left[at \,\middle|\, \begin{matrix}(a_P, \alpha_P)\\(b_Q, \beta_Q)\end{matrix} \right] \cdot H_1\left[\beta t^h, \delta t^k\right]\right\}$$

$$\text{(ii)}\quad M\left\{ H_{u+1, v}^{h, 1}\left[yt^{-1} \,\middle|\, \begin{matrix}(1, 1), g_u, h_u)\\(x_v, y_v)\end{matrix} \right] \cdot H_{P, Q}^{M, O}\left[at \,\middle|\, \begin{matrix}(a_P, \alpha_P)\\(b_v, \beta_Q)\end{matrix} \right] \cdot H_1\left[\beta t^m, \delta t^k\right]\right\}$$

where, $L\{f(t)\}$=Laplace transform of $f(t)$,
$M\{f(t)\}$=Mellin transform of $f(t)$,
$H(x, y)$=Double H-function defined by Mittal and Gupta,
and $H_1(x, y)=H(x, y)$ when $n_1=0$.

52. On (J, p_n) Summability of Fourier Series—*S.K. Varma and S. N Agarwal* (*G.C.D. University, Bilaspur*).

We have proved a theorem on (J, p_n) summability of a Fourier series under conditions on $\{p_n\}$ weaker than those of Khan (Proc. Edinburgh Math. Society 1972).

In fact we shall prove the following theorem :

THEOREM. If $\int_0^t |\phi(u)|\, du = o\left(\frac{p(1-t)}{p'(1-t)}\right)$ as $t \to +0$

and $\int_t^\delta \frac{|\phi(u)|}{u}\, du = o\,(p(1-t)), \qquad (t \to +0)$

for any arbitrary δ, $0<\delta<\pi$, holds, then the Fourier series of $f(\theta)$ is summable (J, p_n) to S at θ_0 where $\{p_n\}$ decreases steadily to zero.

53. The He_n-Transform of the Right-Sided Distributions—*K. C. Prasad* (*Jamshedpur Co-operative College, Jamshedpur*).

The He_n-transform of a right sided distribution is defined. Necessary restrictions on the parameters involved are obtained. Two theorems have been stated and proved. A lemma has been stated and proved and with its help an Analyticity theorem has been established. Finally under the heading remark, He_n-transform reduces to Laplace transform when parameters are suitably chosen.

54. Generalized Laplace Transforms of Distributions—*G. L. N Rao* (*Jamshedpur Co-operative College, Jamshedpur*).

In 1944, T. Carleman was the first author to consider generalized Fourier transform of functions. In 1959, R.A. Kunze studied generalized Fourier transform of functions on Locally compact abelian groups. Later in 1968, this work was extended to distributions by K.I. Gross. In the present paper, the author discusses this problem with reference to the Laplace transform.

55. **An Integral Transform of Generalized Functions**—*Anil Kumar Mahato and K. M Saksena* (*Ranchi University, Ranchi*).

In this paper a generalisation of Laplace transforms, involving Weber's parabolic cylinder function in the kernal is extended to generalized functions. A testing function space is constructed, properties of the testing function space and its dual are studied, transformable generalized functions are defined and an analyticity theorem is proved for the generalized integral transform.

56. **Certain Expansions Involving Basic Hypergeometric Functions of Two Variables**—*Devendra Kandu* (*University of Gorakhpur, Gorakhpur*).

In this paper an attempt has been made to establish certain expansions involving basic hypergeometric functions of two variables. The main results are of the type

$$\sum_{r=0}^{n} \frac{(-1)^r\, q^{\frac{1}{2}r(r-1)}\, \pi(b-r)}{[q]_r\, [q^{1+n}]_{-r}\, \pi(a-r)} \cdot \Phi\left[\begin{matrix}(a_\lambda),\ \triangle(m,\ a-r\ ;\ q^m) : (\beta_\mu),\ (\beta'_\mu); \\ (\gamma_\nu) :\ (\rho_\sigma),\ (\rho'_\sigma)\end{matrix}\ x,\ y\right]$$

$$= \frac{\pi(b)\,[b-a]_n}{\pi(a)\,[1-a)_n}\, \Phi\left[\begin{matrix}(a_\lambda),\ \triangle(m,\ 1+a-b\ ;\ q^m),\ \triangle(m,\ a-n\ ;\ q^m) : (\beta_\mu),\ (\beta'_\mu)\ ; \\ (\gamma_\nu) :\ (\rho_\sigma),\ (\rho'_\sigma)\end{matrix}\ x,\ y\right]$$

which are very useful in the theory of basic hypergeometric functions. By means of these results, we can obtain the series expansion of hypergeometric functions. These results include various known results.

57. **Strong Summability of Functions Based on A_λ Summability Methods (III)** *Babban Prasad Mishra and Gunjeshwar Shukla* (*University of Gorakhpur, Gorakhpur*).

Let $s(u)$ be any function which is lebesgue integrable in $(0, U)$ for all (finite) $U>0$ and which is bounded in some right hand neighbourhood of the origin. Integrals of the form $\int_0^\infty$ are throughout to be taken as $\lim_{x\to\infty} \int_0^x$, $\int_0^x$ being a lebesgue integral.

We suppose that, for $\lambda>-1$ and $t>0$,

$$\Phi_\lambda(t)=\frac{t^{\lambda+1}}{\Gamma(\lambda+1)} \int_0^\infty u^\lambda\, s(u)\, e^{-ut}\, du, \text{ if this exists.}$$

With Jakimovski [Quart. Jour. of Maths. Oxford (2), 9, 34, (1958)], we say that the function $s(u)$ is summable (A_λ) to s for some fixed $\lambda>-1$ if $\Phi_\lambda(t)$ exists for all $t>0$ and tends to s as $t\to 0^+$.

In this paper, strong summability based upon (A_λ) methods for functions is defined and some of its properties investigated.

58. On Certain Special Transformations of Poly-Basic Hypergeometric Functions—*Remy Y. Denis* (*University of Gorakhpur, Gorakhpur*).

In this paper an attempt has been made to utilize the following fractional q-derivative of product of two functions

$$D_q^{\lambda}(UV)=\sum_{n\geq 0}\frac{(-)^n\, q^{\frac{1}{2}n(n+1)}\,[-\lambda]_n}{[q]_n}\, D_q^{\lambda-n}\,[U(xq^n)\, D_q^n(V)],$$

valid for $|x|<R$ $[R=\min(R_1, R_2)]$, where $U(x)=\sum_{r\geq 0} a_r x^r m$, $|x|<R_1$, $V(x)=\sum_{r\geq 0} b_r x^r$, $|x|<R_2$ and $D_q^{\alpha} x^{\mu-1}=(1-q)^{-\alpha}\,\pi\begin{bmatrix}\mu-\alpha\\ \mu\end{bmatrix} x^{\mu-\alpha-1}$, to establish the following transformation of poly-basic q-series

$$\Phi\left[\begin{matrix} q:(\alpha_r),\ p:(c)\ ;\ zu \\ q:(\beta_s),\ p:(d)\end{matrix}\right] \qquad \Phi\left[\begin{matrix} q:(a),\ t:(e)\ yv \\ q:(b),\ t:(f)\end{matrix}\right]$$

$$\Phi\left[\begin{matrix} q:(a),\ p:(c)\ ;\ yu \\ q:(b),\ p:(d)\end{matrix}\right] \qquad \Phi\left[\begin{matrix} q:(\alpha_r),\ t:(e)\ ;\ zv \\ q:(\beta_s),\ t:(f)\end{matrix}\right].$$

59. On Finite Sine Transform of Generalized Functions—*M. S. Choudhary* (*Shivaji University, Kolhapur*).

The purpose of this paper is to extend the sine transform of a function defined on the rectangle $I=\{(x, y): 0<x<a,\ 0<y<b\}$ to a class of generalized functions by using the orthonormal series expansion of generalized functions. The inversion and uniqueness theorem have been proved.

60. On Properties of Self-Reciprocal Functions—*Narendra Prasad* (*Osmania University, Hyderabad*).

In this paper, a few properties of self-reciprocal functions, i.e., if $P(x)$ is a Kernal transforming R_{μ} into $R_{\mu+1}$, the $P(x^2, 2)$ becomes $R_{2\mu+\frac{1}{2}}$, has been verified by mean of the examples involving parabolic cylinder functions and complex variable.

61. Trigonometric Series with Semi-Convex Coefficients—*Renu Bala and Babu Ram* (*M.D. University, Rohtak*).

We generalize Young-Kolmogorov Theorem concerning behaviour of cosine series and further obtain necessary and sufficient condition for the convergence of some cosine sums.

62. Applications of Srivastava's Hypergeometric Function of Three Variables in Heat Conduction—*B. N. Dwivedi* (*Atarra College, Atarra, U.P.*)

Singh [Def Sci., Jour. 21, (1971), p. 225–272] evaluated some integrals involving Kampe de Feriet function and one of them has been employed to obtain a solution of a problem in heat conduction given by Bhonsle [Proc. Nat. Acad Sci. India. 36, (1966), p. 359–364].

In this paper, we evaluate an integral involving Srivastava's hypergeometric function of three variables [Proc. Camb. Phil. Soc. 63 (1967) p. 425–429] and its application is shown in solving a problem on heat conduction given by Bhonsle.

63. On Certain Special Transformations of Poly-Basic Hypergeometric Functions—*Remy Y. Denis* (*University of Gorakhpur, Gorakhpur*).

Making use of the following fractional q-derivatives of the product of two functions

$$D_q^{\lambda}(UV)=\sum_{n\geqslant 0}\frac{(-)^n q^{n(n+1)/2}[-\lambda]_n}{[q]_n} D_q^{\lambda-n}[U(xq^n)]\, D_q^{n}(V),$$

where $|x|<R$ $(R=\min(R_1, R_2))$,

$$U(x)=\sum_{r\geqslant 0} a_r x^r,\ |x|<R_1; \qquad V(x)=\sum_{r\geqslant 0} b_r x^r,\ |x|<R_2,$$

$$D_q^{\alpha} x^{\mu-1}=(1-q)^{-\alpha}\pi\begin{bmatrix}\mu-\alpha\ ;\\ \mu\end{bmatrix} x^{\mu-\alpha-1},\ \pi\begin{bmatrix}\alpha\ ;\\ \beta\end{bmatrix}=\prod_{r=0}^{\infty}\frac{(1-\alpha q^r)}{(1-\beta q^r)},$$

we establish the following transformation of generalized nature

$$\Phi\left[\begin{matrix}(\alpha_r)\ ;\ c_1, e_1 : \ldots : c_n, e_n\ ;\ z_1\xi_1, \ldots, z_n\xi_n\\ (\beta_s)\ ;\ d_1, f_1 : \ldots : d_n, f_n\end{matrix}\right]$$

$$\times\ \Phi\left[\begin{matrix}(\gamma_A)\ ;\ a_1, g_1 : \ldots : a_n, g_n\ ;\ y_1\eta_1, \ldots, y_n\eta_n\\ (\delta_B)\ ;\ b_1, h_1 : \ldots : b_n, h_n\end{matrix}\right]$$

$$=\Phi\left[\begin{matrix}(\alpha_r)\ ;\ a_1, e_1 : \ldots : a_n, e_n\ ;\ y_1\xi_1, \ldots, y_n\xi_n\\ (\beta_s)\ ;\ b_1, f_1 \ldots : b_n, f_n\end{matrix}\right]$$

$$\times\Phi\left[\begin{matrix}(\gamma_A)\ ;\ c_1, g_1 : \ldots : c_n, g_n\ ;\ z_1\eta_1, \ldots, z_n\eta_n\\ (\delta_B)\ ;\ d_1, h_1 : \ldots : d_n, h_n\end{matrix}\right]$$

valid for

$$|z_i\xi_i|, |y_i\eta_i|, |y_i\xi_i|, |z_i\eta_i|<1\ (i=1, \ldots, n)$$

and the Φ-function is defined as

$$\Phi\left[\begin{matrix}(\alpha_r)\ ;\ a_1, b_1 : \ldots : a_n, b_n\ ;\ x_1y_1, \ldots, x_ny_n\\ (\beta_s)\ ;\ c_1, d_1 : \ldots : c_n, d_n\end{matrix}\right]$$

$$=\sum_{m_1,\ldots,m_n\geqslant 0}\frac{[(\alpha_r)]_{m_1+\ldots+m_n}[a_1]_{m_1}[b_1]_{m_1}\ldots[a_n]_{m_n}[b_n]_{m_n}x_1^{m_1}y_n^{m_1}\ldots x_n^{m_n}y_n^{m_n}}{[(\beta_s)]_{m_1+\ldots+m_n}[c_1]_{m_1}[d_1]_{m_1}\ldots[c_n]_{m_n}[d_n]_{m_n}[q]_{m_1}\ldots[q]_{m_n}}$$

valid for $|x_iy_i|<1$, $(i=1, 2, \ldots, n)$ and (α_r) and (β_s) stand for sequence of parameters $\alpha_1, \ldots, \alpha_r$ and $\beta_1, \ldots, \beta_s$, respectively.

64. On Generating Functions of a Triple Hypergeometric Series—*M.A. Pathan and Yasmeen* (*Aligarh Muslim University, Aligarh*).

Some generating function for triple series $F^{(3)}$ of Srivastava are obtained. Our results unify a number of results of Manocha [Proc. Cambridge Phil. Soc. 65 (1969), 687-689] and Srivastava [Rend. Circ. Mat. Palermo Ser. II,

16 (1967), 99-115] and [Ann. Polon. Math. 27 (1972), 73-83]. Special cases appear to give new and known generating functions for appell's functions F_1, F_2, F_3, Kampe de Fe-riet's function $F\begin{matrix} p : q ; r \\ l : m ; n \end{matrix}$, generalized Rice polynomials $H_n^{(\alpha, \beta)}$ and Jacobi polynomials $P_n^{(\alpha, \beta)}$.

65. Generating Functions for the Jacobi Polynomials—*M A. Pathan and Yasmeen* (*Aligarh Muslim University, Aligarh*).

The paper deals with a theorem on triple series and certain general expansions of a different character which gives as special cases new results involving the functions of Kampe de Fe-riet and Appell and Jacobi polynomials. In fact, our theorem extends a result of Cohen [Proc. Amer. Math. Soc. 57 (1976), 271-275] on double series and generalizes a number of generating functions for Jacobi polynomials. Also of interest are erronous results (2.10) and (2.13) of Cohen which are corrected here.

66. Representatian Theory of the Lie Group G (0,1) and Generalized Hermite Polynomials—*M.A. Pathan, Sharief Deshmukh and Ishrat Jahan* (*Aligarh Muslim University, Aligarh*).

Using the multiplier representation of the Lie group G (0, 1), we obtain generating functions for the generalized Hermite polynomials $H_n^{(\alpha)}(x)$. A number of generating functions for the Hermite polynomials $H_n(x)$ are deduced as special cases.

67. On Product of a New Class of Generalized Hermite Polynomials—*M.A. Hussain* (*H.D. Jain College, Arrah*) *and S.N. Singh* (*Avadh University, Faizabad*).

This paper is the continuation of the study of generalized even and odd Hermite polynomials initiated by Thakare and Karande (1973). In the present paper we have obtained the product of arbitrary number of generalized Hermite polynomials in terms of similar polynomials. The generating function for the corresponding coefficients is also obtained.

68. The Generalized Laplace Transform of Distributions—*S.K. Akhaury* (*Ranchi College, Ranchi*).

In this paper we shall study the properties of a distributional generalized Laplace transform. We shall first give some classical properties of generalised Laplace transform of functions and prove analyticity and continuity theorems.

69. Translativity of (J, p_n)-Method—*Z.U. Ahmad* (*Aligarh Muslim University, Aligarh*).

In the present paper the following theorem has been established.

Theorem. The (J, p_n)-method is b-translative if and only if

$$\sum_{n=0}^{\infty} |p_n| \leqslant K, \quad \text{and} \quad \sum_{n=0}^{\infty} p_n = +\infty.$$

Some known results [*e.g.* Borwein, Proc. Camb. Phil. Soc. 1957] have been deduced from this theorem.

70. **On Radius of Close-To-Convexity of Univalent Functions**—*S.R. Kulkarni* (*Willingdon College, Sangli*).

Let $S(\alpha, \beta, \xi)$ be the family of univalent functions in $E=\{z : |z| < 1\}$ satisfying the conditions

$$\left|\frac{zf'/f-1}{[2\xi(zf'/f-\alpha)-(zf'/f-1)]}\right| < \beta,$$

where $\beta \in (0, 1]$, $1/2 \leqslant \xi \leqslant 1$, $0 \leqslant \alpha < 1/2\xi$. In this paper we determine the radii of close-to-convexity of several integrals involving members of $S(\alpha, \beta, \xi)$ and polynomials of degree n whose all the zeros lie outside or on the unit circle.

71. **A Note on the Dirichlet Space D_α**—*Rajiv K. Srivastava* (*Avadh University, Faizabad*).

This paper deals with the Dirichlet space D_α of analytic functions on the open unit disc. We first study some topological properties of D_α, viz., closure theorems, bounded subsets. D_α is then provided a Banach algebraic structure. Topological zero divisors and quasi-invertible elements of this algebra have been investigated. In the last section we have studied multipliers from H^p to D_α.

72. **α-Convexity and α-Close-to-Convexity Preserving Integral Operators**—*K.S. Padmanabhan, R. Parvatham and T.N. Shanmugam* (*Anna University, Madras*).

Let E be the open unit disc in C and $H(E)$ be the class of all functions f holomorphic in E. Let M_α be the Mocanu class of α-Convex functions defined on E. Let $P(\alpha)$ be the class of α-close to-convex functions defined as follows : $P(\alpha)$ is the class of all functions $f(z)$ regular in E, $f(0)=0=f'(0)-1$;

$$\frac{f(z)}{z}\frac{f'(z)}{} \neq 0 \text{ in } E \text{ satisfying}$$

$$\int_{\theta_1}^{\theta_2} e\left\{\alpha\left(1+\frac{zf''(z)}{f'(z)}\right)+(1-\alpha)\frac{zf'(z)}{f(z)}\right\}d\theta \geqslant -\pi$$

whenever $0 < \theta_1 < \theta_2 < \theta_1 + 2\pi$, $z=re^{i\theta}$, $r<1$.

This class was studied by K.S. Padmanabhan and R. Parvatham. Let $g \in H(E)$ be such that $g(0)=g'(0)-1=0$ and $\frac{g(z)\,g'(z)}{z} \neq 0$ in E, We consider the integral operator $A_g(f)=F$, defined by

$$F(z)=A_\alpha(f)=\left\{\frac{\left(C+\frac{1}{\alpha}\right)}{g^c(z)}\int_0^z g^{c-1}(t)g'(t)f^{1/\alpha}(t)\,dt\right\}^\alpha$$

and obtain conditions on g such that $F \in M_\alpha$ (resp. $P(\alpha)$) whenever $f \in M_\alpha$ (resp. $P(\alpha)$. Also we determine conditions on g so that $F \in S^*$—the class of star like functions, whenever $f \in S^*$. Finally we also give an application of the above result.

Our results generalise various results in the literature especially the results due to K.S. Padmanabhan and R. Bharathi, Stephen Ruseshweigh and P.T. Mocanu.

73. On the Fourth Coefficient of Alpha-Spiral-Convex Functions—*M.S. Kasi, (Loyola College, Madras).*

$f(z)=z+a_2z^2+a_3z^3+\ldots$, an analytic function in the unit disc $E=\{z: |z|<1\}$ is called an alpha-convex-spiral function if

$$\operatorname{Re}\left[(e^{i\beta}-\alpha\cos\beta)\frac{zf'(z)}{f(z)}+\alpha\cos\beta\left(1+\frac{zf''(z)}{f'(z)}\right)\right]>0,$$

$f(z)f'(z)/z\neq 0$, $z \in E$, α real, β fixed and $|\beta|<\pi/2$.

If $w(z)=c_1z+c_2z^2+c_3z^3+\ldots$ is analytic in E with $|w(z)|<1$ in E, using the sharp estimate for the functional $|c_3+\mu c_1c_2+\nu c_1^3|$ where μ and ν are arbitrary complex numbers, we deduce the estimate for $|a_4|$ in the class of alpha-spiral-convex functions.

74. On the Distribution of Values of Differential Polynomials—*Subhas S. Bhoosnurmath (Belgaum Campus, Karnataka University, Belgaum).*

If f is a meromorphic function satisfying $\bar{N}(r,f)+\bar{N}(r,1/f)=S(r,f)$ and P is a homogeneous differential polynomial in f which does not reduce to a constant, it is shown that the order of P equals the order of f and $\bar{N}(r,P)+\bar{N}(r,1/P)=S(r,P)$ so that $\Theta(a,p)=0$ for all a, $a\neq 0,\infty$. The impossibility of certain identites between meromorphic functions is deduced improving earlier results of K.L. Hiong.

75. Hadamard Product of Certain Starlike Functions—II—*Vinod Kumar (Christ Church College, Kanpur).*

The author establishes a theorem concerning the Hadamard product of certain starlike and convex functions. The theorem and its applications improve some recent results due to Shigeyoshi Owa [Tamkang J. Math. 14 (1983), 15-21].

76. Jakubowski Convex Integral Operators—*Vinod Kumar and S.L. Shukla (Janta College, Bakewar, U.P.).*

Let $K(m,M)$ denote the class of functions $f(z)=z+\sum_{n=2}^{\infty}a_nz^n$ regular and satisfying $|1+zf''(z)/f'(z)-m|<M$ in $|z|<1$, where $|m-1|<M\leqslant m$. Recently,

R.K Pandey and G.P. Bhargava [Bull. Austral. Math. Soc. 28 (1983)] have shown that if $f \in K(m, M)$, then the function $F(z)=\int_0^z \{f'(u)\}^\alpha \, du$ also belongs to $K(m,M)$ provided α is a complex number satisfying the inequality $|\alpha| \leqslant (1-b)/2$, where $b=(m-1)/M$. In this paper, by taking $f(z)=\int_0^z (1-bu)^{-(\alpha+b)/b} \, du$, where $a=(M^2-m^2+m)/M$, $m=\cdot 7$, $M=\cdot 6$ and $z=-\cdot 8$, we show that their inequality is wrong. We obtain the corresponding results when α is real and complex number respectively. Then we show that $F \in K(m,M)$ provided α is a complex number satisfying $|\alpha| \leq (M-|m-1|)/(M+|m-1|)$. The bound is sharp, and improves and corrects the above mentioned result of Pandey and Bhargava.

77. A Note on a Theorem of Iyer—*N.S. Bhave and T.T. Raghunathan* (*University of Poona, Pune*).

The space of integral functions of V.G. Iyer (1948) was generalised by R.S. Kankurikar (1978) to a space of functions which he denoted as $A(X, C, s)$. In this note we prove that the subspace $A(X, C, s)$ consisting of functions having at most countable number of zeros is of first category. This generalises the corresponding theorem of Iyer.

78. Subclasses of Spiral-Like Functions—*Shashikala D. Patil* (*Karnatak University, Dharwad*).

Let S be the class of functions analytic and univalent for $|z|<1$ and such that each $f \in S$ satisfies $f(0)=f'(0)-1=0$. For, $\alpha \geqslant 0$, M_1, M_2 and λ real such that $|\lambda| < \pi/2$, $M_1+M_2 \geqslant 1$, $M_2 \leqslant M_1 < M_2+1$, we say that $f \in S(\lambda, M_1, M_2)$ if

$$|K(\lambda, \alpha, f(z))-(M_1 \cos\lambda + i \sin\lambda| < M_2 \cos\lambda$$

where $K(\lambda, \alpha, f(z))=(e^{i\lambda}-\alpha\cos\lambda)\frac{zf'(z)}{f(z)}+\alpha\cos\lambda\left(\frac{zf''(z)}{f'(z)}+1\right)$. Each $S_\alpha(\lambda, M_1, M_2)$ is a subclass of the well-known class of Spiral-like functions. The following representation theorem is obtained.

The function $f(z) \in S_\alpha(\lambda, M_1, M_2)$ if and only if there exists a function $F(t) \in S_0(\lambda, M_1, M_2)$ such that

$$f(z)=\left[\frac{e^{i\lambda}}{\cos\lambda}\int_0^z F(t)^{\frac{e^{i\lambda}}{\alpha\cos\lambda}} t^{-1}\, dt\right]^{\alpha\cos\lambda\, e^{-i\lambda}}.$$

In addition, bounds on the functionals $|a_3-\mu a_2^2|$,

where $f(z)=z+\sum_{n=2}^{\infty} a_n z^n$ are obtained.

79. Linear Combination of Functions of Bounded Boundary Rotation of Order α—*Prakash G. Umarani* (*S. Nijalingappa College, Bangalore*).

For $\alpha \in [0, 1)$ and $k \geq 2$, let $V_k(\alpha)$ denote the class of regular functions $f(z)$ satisfying $f(0)=0$, $f'(0)=1$ and $f'(z) \neq 0$ in the unit disk E such that

$$\int_0^{2\pi} \left| Re\left(1+\frac{zf''(z)}{f'(z)}\right)-\alpha \right| d\theta \leq k\,(1-\alpha)\,\pi$$

for all $z=re^{i\theta} \in E$. In this paper we have determined the disk in which the linear combination of f and g in $V_k(\alpha)$ is convex, generalizing an earlier result due to H. Silverman and E. Silvia (On linear combination of convex functions of order α, to appear in Rev. Roum. De Math.).

80. On a Class of Univalent Functions with Negative Coefficients—*S.M. Sarangi* (*Karnatak University, Dharwad*) *and M.R. Krishna Murthy* (*University of Agricultural Sciences, Dharwad Campus*,.

Let $f(z)=a_1 z-\sum_{n=2}^{\infty} a_n z^n \quad (a_n \geq 0,\ a_1 > 0)$ be analytic in a unit disk E.

Let $Q(\alpha)$ denote a class of functions $f(z)$ satisfying $\mathrm{Re}\,\frac{f'(z)}{a_1} > \alpha\ (0 \leq \alpha < 1)$ for $z \in E$, and T a class of functions $f(z)$ that satisfy $f(z_0)=z_0\ (0<z_0<1)$. The subclass $Q(\alpha, z_0)=Q(\alpha) \cap T$ is considered and coefficient inequalities, distortion theorems, radius of convexity and closure theorems are obtained for this class.

81. n^{th} Differential Formulae for a Multivariate Polynomial System—*B.D. Agarwal and Somohita Mukherjee* (*Banaras Hindu Univ., Varanasi*).

Earlier, we have defined a generalized polynomial system $S_n^A\{(x_m), y\}$, with the help of a generating relation involving the Lauricella function F_A viz.

$$\sum_{n=0}^{\infty} S_{n\ ;\ \nu,\ (r_m\ ;\ 1),\ (c_m)}^{A\ ;\ m,\ a,\ (b_m),\ (\mu_m)} \left\{(x_m), y\right\} t^n$$

$$= e^{\nu v t}\, F_A\left[\begin{matrix} a, (b_m) \\ (c_m) \end{matrix} ; \mu_1 x_1\ t,\ \mu_2 x_2\ t\, r^2, \ldots,\ \mu_m x_m\ t\, r^m\right]$$

In the present paper an attempt has been made to express our polynomial set as the n^{th} derivative of some functions. Several new and known results have also been deduced therefrom.

82. On Some Classes of Meromorphic Functions—*G.P. Bhargava, S.K. Srivastava and R.K. Pandey* (*P.P.N. College, Kanpur*).

In this paper we obtain some classical results by using the general integral operators which transform $\Gamma(m, M)$ a subclass of meromorphic starlike functions, $\Gamma^c[A,B]$ meromorphic close to convex functions and

$\Gamma^{\lambda}[A, B]$ meromorphic spiralike function into itself. Our results generalize some known results of Dwivedi, Bhargava and Shukla (1980) and Goel and Sohi (1982).

83. On Convolutions of Functions belonging to Certain Sub-classes of Univalent Functions—*S. Radha, (J.B.A.S. Women's College, Madras).*

Let $H(E)$ denote the class of functions holomorphic in $E=\{z\epsilon C : |z| < 1\}$.

If $f(z)=\sum_{j=0}^{\infty} a_j z^j$ and $g(z)=\sum_{j=0}^{\infty} b_j z^j$ are in $H(E)$ then by $f(z)*g(z)=\sum_{j=0}^{\infty} a_j b_j z^j$ we denote the convolution of f and g. We define the differential operator

$D^n f(z)=\overbrace{(k * k * \ldots * k}^{n \text{ times}} * f)(z)$, $f \epsilon H(E)$, $k(z)=z(1-z)^{-2}$ and $n \epsilon N_0=N^+ \cup \{0\}$. We have $D^0=f(z)$; $D^1 f(z)=zf'(z)$ and $D^n f(z)=D\{D^{n-1}(z)\}=\sum_{j=1}^{\infty} j^n a_j z^j$.

When f and $g \epsilon H(E)$ then g is said to be subordinate to $G[g(z) \prec G(z)]$ in E if $G(z)$ is univalent in E, $g(0)=G(0)$ and $g(E) \subset G(E)$. In what follows, we assume that $h(z)$ is a convex univalent function with $h(0)=1$ and $Re\, h(z)>0$ in E

In this paper we study the two new subclasses of $A=\{f \epsilon H(E) : f(0)=f'(o)-1=0\}$, $S_n(h)$ and $C_n(h)$ which generalise the class S^* of starlike univalent functions of C of close to convex functions respectively. Let $S_n(h)$ denote the class of functions $g(z)$ in A such that $\frac{D^{n+1} g(z)}{D^n g(z)} \prec h(z)$, $z \epsilon E$. $C_n(h)$ denotes the class of functions $f(z)$ in A such that $\frac{D^{n+1} f(z)}{D^n g(z)} \prec h(z)$, $z \epsilon E$, where $g(z) \epsilon S_n(h)$. Here we show that these classes are closed under Bernardi operator, namely, $F(z)=\frac{c+1}{z^c}\int_0^z t^{c-1} f(t)\, dt$, where $c \epsilon C$ with $Re\, c \geqslant 0$ and convolution for $n \geqslant 1$, and establish the inclusion relation $C_{n+1}(h) \subset C_n(h)$ $\forall$ $n \epsilon N_0=N^+ \cup \{0\}$. Further we obtain the cofficient estimate $|a_j| \leqslant \frac{1}{j^{n-1}}$, $j = 2, 3, \ldots$ for $f(z)=z+\sum_{j=2}^{\infty} a_j z^j \epsilon C_n(h)$ which enables us to prove that $\bigcap_{n \epsilon N_0} C_n(h)=\{id\}$, the identity function.

84. On α-Starlike and α-Close-To-Convex Functions with Respect to n-Symmetric Points.—*R. Parvatham (University of Madras, Madras) and S. Radha, (J.B.A.S. Women's College, Madras).*

We define the class $K_n(a\,;h$ the class of α-starlike functions with respect to n symmetric points consisting of $f(z)=z+a_2 z^2+\ldots$ astisfying $\frac{f(z)\,f'(z)}{z} \neq 0$ in E and $\frac{\alpha z\{zf'(z)\}'+(1-\alpha)\, zf'(z)}{\alpha z f'_n(z)+(1-\alpha)\, f_n(z)} \prec h(z)$, $\alpha \geq 0$, where $h(z)$ is

given convex univalent function in E with $h(0)=1$ Re $h(z)>0$ in E and $f_n(z)=1/n \sum_{j=0}^{n-1} \omega^{-j} f(\omega^j z)$ with $\omega=\exp(2\pi i/n)$. In this paper we prove theorems which establish that $K_n(\alpha; h)\subset K_n(0; h)$, $K_n(\alpha; h)$ is closed under an integral operator and the coefficient estimate for this class. Also we define two classes $C_n(\alpha;h)$ and $CS_n(\alpha;h)$ of functions and investigate the properties of these classes.

85. **A Class of Functions defined by Using Hadamard Product**—*O.P. Ahuja (University of Papua New Guinea, Papua New Guinea) and Shigeyoshi Owa (Kinki University, Higashi, Osaka, Japan).*

We say that a function $f(z)=z+\sum_{n=2}^{\infty} a_n z^n$, analytic in the unit disk Δ belongs to the class $P_\alpha(\beta, \gamma)$ if $f(z)$ satisfies the condition $|((f*S_\alpha(z))'-1)/[(f*S_\alpha(z))'+(1-2\beta)]|<\gamma$ for some $\beta(0\leqslant\beta<1)$, $\gamma(0<\gamma\leqslant 1)$, and for all z in Δ, where $f*S_\alpha(z)$ is Hadamard product of $f(z)$ and $S_\alpha(z)=z/(1-z)^{2(1-\alpha)}$. And we denote by $P_\alpha[\beta, \gamma]$, the class obtained by taking intersection of $P_\alpha(\beta, \gamma)$ with the class T consisting of functions $f(z)$ whose nonzero coefficients, from the second on, are negative. The object of the present paper is to determine extreme points, coefficient inequalities, distortion theorems, and radii of starlikeness and convexity for functions in $P_\alpha[\beta, \gamma]$. Further, we give the distortion theorems for fractional integral and derivative of functions in the class $P_\alpha[\beta, \gamma]$.

86. **Some Bilinear and Bilateral Generating Functions**—*M.A. Pathan and Ch. Wali Mohammad (Aligarh Muslim University, Aligarh).*

Three theorems in terms of single and double Laplace and beta integrals are given. These theorems help us in obtaining bilinear and bilateral generating functions of general double and triple hypergeometric series. Our theorems and their applications extend and unify the results of Carlitz [Bull. Un. Mat. Ital. (3) **18** (1963), 87–89], Halim and Al-Salam [Duke Math. J. **30** (1963), 51-62], Manocha [Proc. Camb. Philos. Soc. **63** (1967), 457-499], Manocha and Sharma [Ann. Soc. Sci. Bruxelles Ser. I, **80** (1966), 73–86], Mathur [Bull. Cal. Math. Soc. **70** (1978), 221–227], Saran [Pacific J. Math. **35** (1970), 783–786] and [Indian J. Pure Appl. Math. **3** (1) (1972), 12–22] and Sharma and Mittal [Proc. Camb. Philos. Soc. **64** (1968), 691–694].

87. **The Solution of Some Dual Equations With an Application to a Problem of Elasticity**—*T.N. Trivedi (V.S.S.D. College, Kanpur) and S.S. Pandey (D.A.V. College, Kanpur).*

In this paper dual equations involving the inverse of Naylor's Mellin type transforms have been solved. An application of these dual equations has been shown in the crack problem of infinite elastic solid with circular hole.

88. **Abelian Theorems for Distributional Generalized A-Transform**—*V.K. Chaturvedi and A.N. Goyal (University of Rajasthan, Jaipur).*

The paper consists of the Initial and Final Value Theorems and their distributional extensions for the generalized A-transform defined by the authors (Vijnan Parishad Anusandhan Patrika, 1983).

(D) FUNCTIONAL ANALYSIS

89. **Fixed Points for Four Maps in a 2-Metric Space**—*I.H.N. Rao and K.P.R. Rao* (*Andhra University, Waltair*).

In this paper we prove a coincidence theorem for four nearly densifying self maps on a complete 2-metric space. We also prove a unique common fixed point theorem for four maps by assuming the asymptotic regularity type condition on four maps.

90. **A Generalized Fixed Point Theorem**—*U.N. Singh and S B. Singh* (*Magadh Universty, Bodh Gaya*).

In this paper we have tried to prove a fixed point theorem for six points and ten constants in uniform space by using the technique of Banach contraction principle. Incidentally our results generalize and unify the results of those given by Pittnauer, and Das and Sharma.

91. **Fixed Point Theorem in Banach Space**—*U N. Singh and A.K. Singh* (*Magadh Universty, Bodh Gaya*).

Here we have tried to generalise the results of Kirk [Proc. Amer. Math. Soc. 18 (1970)], Ray [Indian J. Pure Appl. Math. 8 (1977] and Woodword [Notices Amer. Math. Soc. 18 (1971)] by the help of fixed point theorems.

92. **A Common Fixed Point Theorem**—*Tapasundar Bandyopadhyay* (*Visva-Bharati University, Santiniketan*).

The object of the present paper is to generalize a result of Fisher [Math. Sem. Notes, 1979] on common fixed point of two continous commuting self maps of a bounded complete metric space by replacing the commutativity condition by a more general condition. The iterative sequence used for proving the theorem is different from that of Fisher.

93. **Analytic and Harmonic Structure in the Carrier Space of a Real Function Algebra**—*S. Arundhathi and S.H.Kulkarni* (*I.I.T, Madras*).

The relationship between the harmonicity and analyticity of a continuous map from the open unit disc to the carrier space of a real function algebra is investigated. In particular, the following results are proved :

(i) Let X be a compact, Hausdorff space, $\tau : X \to X$ a homeomorphism such that $\tau o \tau$ = identity on X, A a real function algebra on (X, τ), U the open unit disc in the complex plane, Φ_A the carrier space of A and $Y = \{\phi \in \Phi_A : \phi = \bar{\phi}\}$, the real locus of Φ_A. Suppose that $F : U \to \Phi_A$ is a continuous map such that Re $(\hat{f}oF)$ is harmonic for every f in A. (Here $\hat{f}$ denotes the Gelfand transform of f). Then in every connected component of $U \sim F^{-1}(Y)$, $\hat{f}oF$ is analytic for all f in A or $\hat{f}oF$ is analytic for all f in A.

(ii) If X is a compact plane set which is symmetric with respect to the real-axis and which has a connected interior, $\tau : X \to X$ is complex conjugation and if A is a real function algebra on (X, τ) such that Re f is harmonic for every f in A, then f is analytic for every f in A or $\bar{f}$ is analytic for every f in A in interior of X.

94. A Representation of a Stieltjes Transformable Generalized Function—*Snehal Dange and M S. Chaudhary (Shivaji University, Kolhapur).*

In this paper the spaces $\bar{J}_{c,\,d}$ and its dual space $\bar{J}'_{c,\,d}$ are given. An extension of the classical Stieltjes Transfom to a certain space of generalized functions is given and a structure formula for a Stieltjes Transformable generalized functions is obtained which shows that every element of the dual space $\bar{J}'_{c,\,d}$ is the linear combination of the finite order distributional derivative of continuous functions.

95. Approximation of Invariant Operators in a Banach Space—*G S. Pandey (Vikram University, Ujjain).*

Let G be the circle group and let $L_1(G)$ be the space of all those functions f on G such that

$$\int_G |f|\,dt<\infty.$$

Let B be a dense linear subspace of $L_1(G)$, which is translation invariant. We also assume that B is a Banach space with respect to a norm $\|.\|_B$ and satisfying the conditions :

(*i*) $\|f\|_{L_1(G)}\leqslant \|f\|$, $f\in B$.

(*ii*) The norm is translation invariant.

(*iii*) The functions in B translate continuously.

(*iv*) B is closed under multiplications by the characters of G.

We denote by L the Banach algebra of bounded linear operators on B with respect to the norm $\|.\|_L$ and by L_i the set of invariant operators in L.

An operator $T\in L$ is said to be almost invariant iff

$$\lim_{t\to 0}\|TR_t-R_tT\|_L=0.$$

K. De Leeuw (1965) was the first to have studied in detail the harmonic analysis for operators on the circle group G. In the present paper we prove the following theorems :

THEOREM 1. *If T is an almost invariant operator, then $S(T)$ is summable to T in the operator norm, provided $\{p_n\}_{n=0}^{\infty}$ is a non-negative and non-increasing sequence such that*

$$\sum_{k=0}^{n} P_k/(k+1)=o(P_n),\ P_n=p_0+p_1+\ldots+p_n,\ P_n\neq 0.$$

THEOREM 2. *If*

$$\|TR_t-R_tT\|_L=o\,(t^{\alpha})\ ;\ 0\leqslant\alpha<1,$$

then

$$\|\sigma_n^{\beta}(T)-T\|_L=o\,(n^{-\alpha})$$

for all $\beta>\alpha$, where $\sigma_n^{\beta}(T)$ denotes the Cesaro mean of order β of the series $S(T)$.

we also derive a number of new results from the above theorems.

96. **On Quasi *-barrelled Spaces**—*S.G Gayal* (*Arts, Science, Commerce College, Rahuri*).

In this paper, a new class of locally convex spaces, called quasi-*-barrelled spaces is introduced. These spaces are characterized by : A locally convex space E is Quasi-*-barrelled if every bornivorous *-barrel in E is a neighbourhood of 0 in E This class of spaces is a generalization of quasi-barrelled spaces and *-barrelled spaces (K. Anjaneyulu, Gayal : Jour. Math. Phy. Sci. Madras, 1984). Some properties of quasi *-barrelled spaces are studied. Lastly, one example each of

(*i*) a quasi *-barrelled space which is not quasi-barrelled.

(*ii*) a quasi *-barrelled space which is not *-barrelled,

is given.

97. **Quasisimilarity and k-Quasi-Hyponormal Operators**—*B.C. Gupta* (*Sardar Patel University, Vallabh Vidyanagar*).

An operator A on a Hilbert space is called k-quasihyponormal if $A^{*k}(A^*A-AA^*)A^k \geqslant 0$, k being a positive integer. Conditions under which two quasisimilar k-quasihyponormal operators have equal essential spectra are obtained and several results known for hyponormal operators have been extended. It is proved that a cyclic 1-quasihyponormal operator has trace class self commutator. Finally, a result of Duggal on subnormal operators has been proved for M-hyponormal operators and for certain k-quasi-hyponormal operators.

98. **Some Properties of Functionals in p-Normed Spaces**—*Prem Chandra* (*Vikram University, Ujjain*).

We suppose that X is a p-normed linear space with $p>0$ and $A : X \to C$ is linear. Then we prove the following : THEOREM 1. *Let A be bounded and let $A \neq 0$. Suppose $M=\{x \in X : A(x)=\alpha\}$, where α is a fixed scalar, and d is the distance of the hyperplane M from the point $a \in X$, then $d=|\alpha-A(a)|^p/\|A\|^p$.* THEOREM 2. *$A$ is continuous if and only if the set*

$$F=\{x \in X : A(x)=\alpha\}$$

is closed.

99. **On Rotundity of Convex Functions**—*P. Govindarajulu* (*Sri Venkateswara University, Tirupati*).

Let f be a continuous convex function on a locally convex space X with $f(\theta)=0$. For each non-empty closed set V of X and x in X, we define the set

$$D(x, V)=\left\{z \in X : \inf_{v \in V} p_r(z-v)=p_r(z-x)+\inf_{v \in V} p_r(x-v)\right\}$$

where p_r is a Minkowski guage of the sublevel set S_r of f, r a positive real number. In this paper we characterize the Rotundity of convex function f in terms of the set $D(x, V)$.

100. **A note on contractors in random normed spaces**—*A. Chitra and P.V. Subrahmanyam* (*I.I.T., Madras*).

In this note we extend the concept of contractors in the setting of random normed spaces introduced by Serstnev (1963) and obtain an existence theorem generalizing that of Altman.

101. Generalization of Ko's Fixed Point Theorems—*R. Baskaran* (*A.M Jain College, Madras*).

Ko has obtained an interesting generalization of fixed point theorems due to Belluce and Kirk for nonexpansive multimaps based on the concept of convex set-valued maps. We obtain two coincidence theorems generalizing the fixed point theorems due to Ko, and Belluce and Kirk We also provide examples illustrating our main theorems.

102 A Functional Analytic Study of Hilbert's Thirteenth Problem—*V.L.N. Sarma* (*St. Lawrence University, Canton, New York*).

Hilbert's 13th problem implicitly raised the question whether continuous real functions of k (>2) real variables can be represented in terms of continuous functions of one variable using a finite number of additions and superpositions. A 1958 theorem of Kolmogorov (refined by Sprecher and by Lorentz), which provides an affirmative answer, can be stated in the form : Given rationally independent real numbers, λ_p $(1 \leqslant p \leqslant k)$ there exist continuous functions, ϕ_q $(1 \leqslant q \leqslant 2k+1)$ on $I=[-1, 1]$ into I such that for each continuous real function, f, on I^k, there is a continuous real function, g, on $[-k,k]$ for which a representation

$$f(x_1, \ldots, x_k) = \sum_q g\left(\sum_p \lambda_p \phi_q(x_p)\right)$$

holds. It is not known whether a choice of λ_p's and ϕ_q's exists for which the map, $f \to g$ can be made linear and continuous However, if f is restricted to be real analytic carrying the l^1 - norm, it can be shown (using Kolmogorov's theorem) that such a bounded linear map on l^1 into $C[-k,k]$ is possible. A method of search for such a map provides a means to an independent functional analytic proof of Kolmogorov's theorem. For a general such map T we average $\left| f(x_1, \ldots, x_k) - \sum_q Tf\left(\sum_p \lambda_p \phi_q(x_p)\right) \right|^2$ first over x_p's and then over the space of f's (with respect to the Eberlein measure in l^1) and call this final average $\sigma^2(T)$. Conditions on the λ's and the ϕ's are obtained (by me and my student, Dr. Shantanu Bhattacharya) for inf $\sigma^2(T)$ to vanish.

103. Common Fixed Point Through A New Rational Inequality—*I.H N. Rao and K.P.R. Rao* (*Andhra University, Waltair*).

In this paper we prove two unique common fixed point theorems for a pair of maps satisfying a new rational inequality. We also observe that two theorems of Sharma and Yule (Bull. Cal. Math. Soc. 1984) are not valid and suggest some modifications.

104. On Semi m-Normed Spaces—*S C. Gupta, A.H. Siddiqi and A. Siddiqi* (*Aligarh Muslim University, Aligarh*).

The concept of m-normed space and m-metric space was introduced and investigated by S Gahler in a series of papers (Math. Nachr 40 (1969), 165-189, 229-264, 41 (1969), 23-36]. The non-archimedean analogue of these spaces was studied by A.H. Siddiqi, S.C. Gupta and A. Siddiqi (Preprint).

Recently the concept of semi 2-normed spaces was introduced by R W. Freese and S Gahler [Math Nachr. 165(1982) 151-161]. In the present paper semi m-normed spaces are studied.

We mention here one of our results.

Theorem : In a semi-m-normed space L, the following statements are equivalent

1. For all $a, a_1, a_2, \ldots, a_m \in X$ with $a = (a_1, a_2, \ldots, a_m)$

$$V(a_{a_j} \to a_j + a) \leq V(a) + V(a_{a_i} \to a)$$

2. For all $a, a_1, a_2, \ldots, a_m \in X$ with $a = (a_1, a_2, \ldots, a_m)$

$$V(a_m \to a_i + a) \leq V(a) + \sum_{i=1}^{m} V(a_{a_i} \to a_i)$$

For each i.

3. For all $a, a_1, a_2, \ldots, a_m \in X$ with $a = (a_1, a_2, \ldots, a_m) \in X^m$

$$\lambda(a) \leq \sum_{i=1}^{m} \lambda(a_{ai} \to a),$$

where λ is natural semi-m-metric on X.

105 On Linear Functionals—*K.C. Varshney (Aligarh Muslim University, Aligarh).*

In this paper, we have established two results on linear functionals which are not necessarily continuous. Theorem 1. Let f be a linear functional defined on a Banach space X. Then there is a positive number C depending on f, such that for any element $x \in X$ there exists a sequence $\{x_n\}$ in X which strongly converges to x with the property that $|f(x_n)| < C \|x\|$, and furthermore, $\{f(x_n)\}$ is convergent.

Theorem 2. Let f be a linear functional defined on a normed linear space X, whose range is all of R. Then there exists a positive number C such that for any real number $t \in R$ there exists a sequence $\{x_n\}$ in R which converges to t with $t_n = f(x_n)$ and $\|x_n\| < C|t|$. Moreover, $\{x_n\}$ is a Cauchy sequence.

106. On Semi-Banach and Semi-Hilbert Algebras—*B. Musavi (Aligarh Muslim University, Aligarh).*

A new class of algebras, viz 'Semi Banach Algebras' has been introduced. It is defined to be an algebra A whose underlying space is equipped with a complete semi-norm p which is sub-multiplicative, i.e.

$$p(x, y) \leq p(x)\, p(y) \qquad \forall\ x, y \in A$$

Several examples of such algebras, in particular function algebras have been studied in detail, and it has been observed that many results which hold in the setting of Banach algebras cease to hold in case of semi-Banach algebras. A number of examples are constructed to elaborate the situations.

Hilbert algebras have also been generalized to 'Semi-Hilbert Algebras.' Some counter-examples have been constructed to bring out the cases where these algebras behave in a manner different from the classical situations in Hilbert algebras. An important result due to Yood [Hilbert Algebras as Topological Algebras, Arkiv for Mat. Vol. 12, No. 1, 130-151, May (1974)] has been extended to semi-Hilbert algebras.

107. Fixed Point Results for Mappings on Uniformly Convex Banach Spaces—*Tanmoy Som (P. U. College, Aizawal, Mizoram).*

In the present paper we have given few common fixed point results for a triple of mappings on a uniformly convex Banach space satisfying a new functional inequality condition of Prasad (1984) type. In another result we have extended the main theorem of Prasad for triple of mappings. In all the results we have weakened the commutativity condition of the mappings by using weakly commuting (w.c.) mappings. Finally, Theorem 1 of Sahani and Bose (1984) is refined by considering w.c. mappings. Our results mainly extend and unify the results of Bose (1978). Iseki (1976), Husain and Sehgal (1976) and Singh and Meade (1977). At the end we have given a few example verifying our generalizations.

(E) DIFFERENTIAL EQUATIONS, INTEGRAL EQUATIONS AND FUNCTIONAL EQUATIONS

108. Oscillation and Non-Oscillation Theorems for Non-linear Volterra Stieltjes Integral Equations *G. V. Ravindranadh Babu (Andhra University, Waltair).*

Sufficient conditions for oscillation and non-oscillation of the non-linear Volterra Stieltjes Integral equation

$$x(t)=f(t)+\int_0^t g[t, s, x(s)]\, du(s)$$

where $f\,[0, \infty) \to R$ continuous and

g : $[0, \infty) \times [0, \infty) \times IR \to R$ continuous and

u : $[0, \infty) \to R$ is continuous and is a function of bounded variation, are obtained in this paper.

109. Oscillation Theorems for Arbitrary Order Functional Differential Equations with Advanced Argument—*R.S. Dahiya (Iowa State University).*

In the present paper we study the oscillatory behavior of solutions of the functional differential equation :

$$(1) \qquad y^{(n)}(t) - a(t)y\,[g(t)] = f(t),\ n \geqslant 2,$$

where

(*i*) $a(t)$ is continuous and positive on $[t_0, \infty)$;

(*ii*) $g(t)$ is non-decreasing continuous function on $[t_0, \infty)$ and such that $g(t) > t$;

(*iii*) $f(t)$ is continuous on $[t_0, \infty)$.

We provide sufficient conditions for the above functional differential equation to be almost oscillatory in the sense that every solution $y(t)$ of (1) is either oscillatory or else it satisfies $\lim_{t\to\infty} | y^{(i)}(t) | = \infty$, $0 \leqslant i \leqslant n-1$ or $\lim_{t\to\infty} y^{(i)}(t) = 0$, $0 \leqslant i \leqslant n-1$. A further condition is given which precludes the possibility that $\lim_{t\to\infty} | y^{(i)}(t) = \infty$, $0 \leqslant i \leqslant n-1$ in the above statement.

These results compliment and extend my earlier results in JMAA, Vol. 91 (1983)] to advanced arguments.

110 On the Bifurcating Solutions of B-Z Reaction Equations - *M R. Kaimal and Mercy Mani P (University of Cochin, Cochin).*

Only a small number of chemical systems exhibits typical non-linear behaviour i.e. non-monotonous dynamics exciting, oscillating or chaotic. Among these the Belousov–Zhabotinski (B–Z) reaction is by far the most studied experimentally and to some extent theoretically. Recent experimental studies of the B-Z reactions have shown that the exhibit phenomena such as Hopf-bifurcations and period doubling bifurcations. In this paper, we investigate the properties of solutions of a suitable model equation, near Hopf bifurcation point. We develop suitable bifurcation formulae to compute the form of oscillations, their amplitudes, their periods and study their stability or lack of it. The effect of diffusion induced stability or lack of it of periodic solutions is also studied here.

111. Some Functional Equations on Topological Groups—*Ravindra D. Kulkarni (University of Bombay, Bombay).*

In [Studia Math. 19 (1960)], Kurepa obtained a solution of D'Alembert's functional equation

$$f(x + y) + f(x - y) = 2 f(x) f(y),$$

when f is a complex-valued function on a Banach Space.

In this paper we prove that Kurepa's theorem holds good when the domain of f is just a *topological vector space over Q* or an abelian topological group G whose dual group $\hat{G}$ is a union of one-parameter subgroups and the natural map τ of G to its second dual $\hat{\hat{G}}$ is continuous.

We also extend Kurepa's results in [(Studia Math. 19 (1960)], regarding other functional equations when the domain is same as the one mentioned above for D'Alembert's functional equation.

112. Parameter Sets for a Class of Boundary Value Problems—*D.Y. Kasture (Marathwada University, Aurangabad).*

Two point boundary value problems for the class of differential equations $\epsilon y'' + g(y) h(y') = 0$, $\epsilon > 0$, are considered with prescribed Dirchlet boundary conditions or mixed boundary conditions. The sets of all values of the parameter ϵ, for which a solution of the problem exists, is called the parameter se for the problem. Under certain conditions on g and h, S is compeletely determined. It is shown that for a class of problems, S does not contain a right neighbourhood of zero.

113. Singularly Perturbed Boundary Value Problems for Differential Equations in Banach Space—*N. Ramanujam (Bharathidasan University, Tiruchirapalli) and V.M. Sunandakumari (University of Cochin, Cochin).*

Two point boundary value problems, described by weakly coupled systems of second order, ordinary differential equations defined in a Banach space with a small parameter multiplying the highest derivative, are studied using second order differential inequalities in Banach space. Estimates for

solutions are obtained which are then utilised to derive results on the limiting behaviour of solutions as the small parameter goes to zero.

114. Singularly Perturbed Initial Value Problems for Differential Equations in Banach Space—*N. Ramanujam* (*Bharathidasan University, Tiruchirapalli and V.M. Sunandakumari* (*University of Cochin, Cochin*).

Initial value problems, described by weakly coupled systems of first order ordinary differential equations defined in a Banach space with a small parameter multiplying the derivative, are studied using first order differential inequalities in Banach Space. Estimates for solutions are obtained which are then utilised to derive results on the limiting behaviour of solutions as the small parameter goes to zero.

115. Kowalevska's Exponents and Painleve Property of Liouville Equation in One Independent Variable—*B.V. Baby* (*Bharata Mata College, Cochin*).

The Kowalevska's exponents and the Painleve property of the single independent variable Liouville equation, $d^2\varphi/d\tau_i^2=\exp(2\varphi)$, where τ_i is x or t, have been studied in this paper. It is found that Kowalevska's exponents are real and rational numbers and so the equation is possibly algebraically integrable. The conditions of the Painleve property are also fulfilled.

116. On non-Linear Integrodifferential Equation with Delay Arguments—*M.A. Hussain* (*Marathwada University, Aurangabad*).

The purpose of this paper is to study the existence, uniqueness, continuous dependence on a parameter and asymptotic behaviour of solutions of a non-linear integrodifferential equations with delay arguments. We use the well known Banach fixed theorem and integral inequality established by Pachpatte to obtain our results.

117. Perturbations of Measure Differential Equations—*K. Satyavani* (*Andhra University, Waltair*).

The effect of small perturbations in the coefficient matrix of a linear measure differential equation or the roughness of such an equation is studied in this paper. The notion of exponential dichotomy plays the crucial role in this study. The following theorem sums up the main results of the paper :

THEOREM : Suppose A and B are continuous matrix valued functions on R^+ and u is a function of bounded variation and right continuous on R^+ and the measure differential equation

$$Dx=A(t)xDu$$

has an exponential dichotomy with constants K, α, projection P and fundamental matrix X.

If $\delta=\sup_{t\geq 0}\|B(t)\|$ then the linear measure differential equation

$$Dy=(A(t)y+B(t)y)\,Du$$

also has an exponential dichotomy provided K, α, u, δ and A are related by the following conditions :

1. $3K\delta \int_0^\infty e^{-\alpha t}\, dV_u(t) > 1$, V_u being the total variation function of u,
2. $K\delta\, V_u(\infty) < 1$
3. $\dfrac{K^2 V_u(\infty)}{1-K\delta V_u(\infty)} < \frac{1}{2}$, and
4. $\delta[(K_1 + TV(X))\, L_1 + (K_1 + TV(X)(\, L_2]\, K_2 V_u(\infty) < 1$

where $K_1 = \underset{t\geq 0}{\text{Sup}} \| X(t) \|$, $K_2 = \underset{t\geq 0}{\text{Sup}} \| X^{-1}(t) \|$, $\| P \| = L_1$, $\| I-P \| = L_2$

and $TV(X)$ denotes the total variation of X.

118. Controllability of Perturbed Nonlinear Delay Systems—*K. Balachandrau (Bharathiar University, Coimbatore).*

Let $J=[t_0, t_1]$ be a bounded interval. Let $h>0$ be a real number. Consider the linear delay system

$$\dot{x}(t) = L(x, u) \qquad \ldots(1)$$

where $$L(x, u) = \int_{-h}^{0} d_s\, A(t, s)\, x\,(t+s) + B(t)u(t).$$

$A(t, s)$ is an $n \times n$ matrix continuous in t uniformly with respect to s in $[-h, 0]$ and of bounded variation in s on $[-h, 0]$ for each t in J and $B(t)$ is a continuous $n \times m$ matrix. Its corresponding perturbed nonlinear system is

$$\dot{x}(t) = L(x, u) + f(t, x(t), x(t-h), u(t)), \qquad \ldots(2)$$

where the function f is continuous in its arguments. Further, the integral is in the Lebesgue-Stieltjes sense.

THEOREM : If the continuous function f satisfy the condition

$$\lim_{|p| \to \infty} \frac{|f(t, p)|}{|p|} = 0$$

uniformly in t in J and if the system (1) is controllable on J, then the system (2) is controllable on J.

119. A General Partial Differential Equation and Its Complete Solution—*I.K. Khanna and Krishna Yadava (Banaras Hindu University, Varanasi).*

The purpose of the present paper is to derive a general partial differential equation for the generalized polynomial $E_n{}^A\ \{(x_m),\ y\}$ and thereafter to discuss its complete solutions which is followed by some applications.

120. Nonoscillation Theorem For A Second Order Nonlinear Functional Differential Equation—*S.K. Nayak (Berhampur University, Berhampur).*

Sufficient conditions have been obtained for the non-oscillation of all bounded solutions of

$$(r(t)\, y'(t))' - p(t)\, y^{\nu}\,(g(t)) = f(t),$$

where r, p, g and f are real valued continuous functions on $[0, \infty)$ such that $r(t)>0$, $\lim_{t\to\infty} g(t)=\infty$, $p(t)$ or $f(t)$ or both are allowed to change sign and $\nu>0$ is a ratio of odd integers.

121. Periodic Boundary Value Problems for an Infinite System of Nonlinear Second Order Differential Equations—*K. Narasimha Reddy* (*Osmania University, Hyderabad*).

We first obtain results on the existence of solutions of periodic boundary value problems for an infinite system of nonlinear second order differential equations by combining fruitfully, the method of upper and lower solutions and the Lyapunov-Schmidt method. Also monotone iterative method is described for the extremal solutions and results on the existence and uniqueness of solutions are obtained.

122. Inequaiities for the Eigen values of powers of functions—*Veeravalli Srinivasan and M. Satyanarayana* (*S.V. University, Tirupati*)

Fink (1969) proved a comparison theorem between the smallest eigen values of the two related differential equations

1. $y''+\lambda py = 0, \quad y(0)=0, y(T)=0$ and
2. $y''+\lambda p^2 Y = 0, \quad y(0)=0, y(T)=0$

In this paper the result of Fink is geneeralised in two ways. In one way the higher order differential equations are considered. In the other way differential equations of second order are considered, but higher powers of p are taken. Combining these two generalisations, comparison theorems for higher order equations with higher powers of p are proved.

123. Effect of a Cubic Non-Linearity on The Motion of a Spinning Axis-Symmetric Body—*Smita D. Naik and K C. Sharma* (*Institute of Armament Technology, Pune*).

The dynamical system of an axis-symmetric body is always stable for its positive values. If bifurcates into stable and unstable branches for negative nonlinear parameter only. Bifurcation happens at $m=-1/\sqrt{108F^2}$ and the stability parameter $\sigma^2=3\left(\frac{2\phi'}{K_2}-1\right)^2$ along these two branches. For the complete stable motion the equilibrium points of the system are bounded from above, given by $\beta=E+\frac{1}{2m}+\frac{mF^2}{2}$, $m>0$ and $\beta=\frac{1}{2m}+\frac{3F^2}{2E}$, $m<0$ under some stipulated conditions.

124. Measure Differential Equations in Hilbert Space—*S. V. Krishna and K. Satyavani* (*Andhra University, Waltair*).

Measure differential equations in Hilbert space are introduced and studied. The solutions are Hilbert space valued measures. They turn out to be absolutely continuous with respect to a known (given) real valued measure. We obtain a successive approximation method of construction of

a solution for such equation and prove existence theorem for the solution of a measure equation involving maximal monotone operators thus laying a foundation for a semigroup theory for measure equation. Stability of such equation is also discussed.

125 **Existence of The Finite Escape Type Solutions**—*J. Sucharitha (University College for Women, Hyderabad).*

In this paper a theorem concerning the existence of the finite escape type solution to the general second order differential equation

$$y''(t)P=(t)\ g\ (y\ (t))+f(t),\ y\ (a)>0,\ y'\ (a)\geqslant 0$$

for $t\in (a.\ \infty)$ is established. In addition, a comparision technique is used existence of the finite escape type solution to the second order Integro-differential equation

$$Y''(t)=f(t)+\int_a^t k(t-s)g(y,s)\ ds,\ t\geqslant a.$$

Lastly, the existence of only unbounded solutions to the Integral Equation of the type

$$x(t)\cdot f(t)-\int_a^t a\ (t,\ s)\ g\ (s,\ x(s))\ ds,\ t>0$$

is also established.

(F) GEOMETRY

126. **Finsler Spaces Associated with Cubic Metric**—*U.P. Singh and B.N. Gupta (University of Gorakhpur, Gorakhpur).*

The notion of cubic Finsler spaces has been introduced by Matsumoto and Numata. In this paper we have defined a metric function $^*L(x, y)$ using the metric $L(x, y)$ of cubic space F_n. The special Finsler space equipped with metric $^*L(x, y)$ is denoted by $^*F_n$. Such a Finsler space is a generalized form of Rander's space which is characterised by the special metric $F=\alpha+\beta$, where α is Riemannian metric and $\beta=b_i(x)\ y^i$. After deriving the fundamental quantities, we have obtained the expression for v-curvature tensor of space $^*F_n$ and established a condition for $^*F_n$ to be S 3-like. The relation between the T-tensor of spaces $^*F_n$ and F_n has been obtained in the paper. Finally, we have studied the effect of conformal transformation on space $^*F_n$.

127. **Some Properties of the Discrete Holometric Space–II**—*A. Vijayakumar, (University of Cochin, Cochin)*

We continue the study of the properties of the discrete holometric space, $H=\{(q^m x_0,\ q^n y_0)\ ;\ m, n\in Z$, the set of integers$\}$, $q\in(0, 1)$ is fixed and (x_0, y_0) is also fixed, $x_0, y_0\neq 0$. Earlier, we have introduced analogues of classical geometric concepts in H, in order to develop a basic geometry of the

discrete plane. In this paper, we introduce the idea of associating a distance matrix for finite subsets of H and obtain the properties of those associated with certain special class of domains Also, the notion of metric content, an estimate for the metric content of an r-set etc. are discussed.

128. Conformal and h-Conformal Transformation in Special Finsler Spaces—*U P. Singh and B N Gupta (University of Gorakhpur, Gorakhpur).*

The notion of conformal theory of Finsler spaces has been introduced by M.S. Knebelman and several other authors In 1976, M. Hashiguchi established the conformal theory of Finsler metrics based on the geometry of Finsler spaces developed by M. Matsumoto. On the other hand, the notion of h-conformal transformation has been introduced by H. Izumi. The purpose of the present paper is to study the effect of conformal and h-conformal transformations on Landsberg, P^*-, P-reducible, semi-P 2-like and P-symmetric Finsler spaces. We have established conditions under which these special Finsler spaces will remain the spaces of the same kind under conformal and h-conformal transformations.

129. On The Geometry of The Very Vast Plane—*T. Thrivikraman (University of Cochin, Cochin).*

In earlier papers, the author has introduced and studied the very long line L_α (for any cardinal numder α) in relation to the notion of K-compactness of Herrlich. Here we call the space $L_\alpha \times L_\alpha$ as the very vast plane and some of its geometric properties are investigated.

130. A Translation Plane of Order 25 With Small Translation Complement—*M.L. Narayana Rao, K. Kuppu Swamy Rao and Vinod Joshi*

Various translation planes of order 25 have so far been constructed and their translation complements determined. These are

1. The fifteen planes reported by Davis
2. The plane reported by Rao and Rao
3. The plane reported by Narayana Rao and Satyanarayana
4. The two exceptional planes reported by Walker
5. The two flag transitive planes reported by Foulser.

All these planes except the two translation planes of Foulser and one of the exceptional planes of Walker are such that atleast one of their matrix representation sets admit non-trivial nuclei.

In this paper we construct a new translation plane of order 25, all of whose matrix representation sets admit only a trivial nucleus. Further the translation complement modulo the sub-group of the scalar collineations is computed as a dihedral group of order 24 and which is the smallest when compared with all planes reported so far The translation complement of this plane divides the sets of ideal points into four orbits of length 4, 4, 6 and 12. From the study of orbit structures we conclude that this plane is not one of the seventeen listed above.

131. Fertility Characteristics of Quadratic Symmetric Forms—*M.N. Ramakrishna Pillai (Trivandrum).*

In the case of the triangle the coordinates of whose vertices are (x_1, y_1) (x_2, y_2), (x_3, y_3) there are 26 quadratic symmetric points. The fertility of quadratic symmetric points depends upon the extent of their structural symmetry. The quadratic symmetric points have two or three terms while there are six, four or two characteristic terms in the numerator determinants. in the coordinates of the point of concurrence of perpendiculars (c.p c.p.) from these points to the sides of the triangle If the number of characteristic terms in the numerator determinant of the c.p c p. is twice the number of terms in the symmetric points, the symmetric form is highly fertile, if it is two less it has medium fertility while it has low fertility if it has the minimum number of terms *viz.* two Of the 26 quadratic symmetric forms 15 have high fertility, 7 are of medium fertility and 4 are of low fertility. These rules also apply to the first and second shift operated points obtained from these points These rules also apply to the first and second shift operated points obtained from these points.

132. Structure of Random Quadratic Symmetric Points of a Triangle and Their Associated Determinants—*M N. Ramakrishna Pillai (Trivandrum).*

In the case, of the triangle ABC the coordinates of whose vertices are (x_1, y_1), (x_2, y_2), (x_3, y_3) there are 26 quadratic symmetric points (q.s.p.) of which seven are random q s p. The coordinates of the random q.s p. of the side BC and the numerator of the coordinates of the points of concurrence of perpendiculars (c.p c.p.) from these points and from points whose coordinates are obtained by cyclically permuting the suffixes of the x and the y coordinates of these points on the sides CA & AB are $D(H_1, y_1)$ and $D(x_1, H_1)$ where H_1 in each case is

(1) $\{(x_1^2 - x_2x_3), (y_1^2 - y_2 y_3)\}\ (x_2^2 x_1 + x_3^2 x_1 + y_2^2 y_1 + y_3^2 y_1)$

(2) $\{(x_1 y_1 - y_2 y_3), (x_1 y_1 - x_2 x_3)\}\ (x_1 x_2 y_2 + x_1 x_3 y_3 + x_2 y_2 y_1 + x_3 y_3 y_1)$

(3) $\{(x_2 y_1 + x_3 y_1 + y_1^2)\ (x_1 y_2 + x_1 y_3 + x_1^2)\} - (x_3^2 y_2 + x_2^2 y_3 + x_2 y_3^2 + x_3 y_2^2)$

(4) $\{(x_1 y_2 - y_1 y_2), (x_2 y_1 - x_1 x_2)\} - (x_2 x_3 y_1 + x_1 y_2 y_3)$

(5) $\{(x_3 y_2 + x_3 y_3 + y_3^2), (x_2 y_3 + x_2 y_3 + x_3^2)\}\ (x_1^2 y_1 + x_1 y_1^2)$

(6) $\{(x_1 y_3 - y_1 y_3), (x_3 y_1 - x_1 x_2)\} - (x_2 x_3 y_1 + x_1 y_3 y_3)$

(7) $\{(x_3 y_3 + x_3 y_3 + y_2^2), (x_3 y_3 + x_3 y_3 + x_2^2)\}\ (x_1^2 y_1 + x_1 y_1^2)$.

The denominator is always $D(x_1, y_1)$. In the case of points 1 and 2 the c.p.c.p. are obtained by multiplying the complementary set of the leading term of the x coordinate by x_1 and of the y coordinate by y_1. In the case of symmetric points 4 and 6 whose terms are assymmetric, the c.p.c.p is obtained from the actual and the symmetric terms of the c.p.p d. (coordinates of points from which perpendiculars are drawn) viz x_1y_2 and x_1y_3 and x_2y_1 and x_3y_1. the c.p. p.d. of 3, 5 and 7 are obtained from those of 2, 4, 6 by substituting for the terms the restricted complementary sets and the complementary products while the c.p.c.p. are obtained by substituting the complementary products for the terms of the c p.c.p. Similar results hold for the first and second shift operated points of these symmetric points.

133. A Class of Non Kaehler Moishezon Manifolds—*M. Siaramayya (University of Hyderabad, Hyderabad).*

Recently the author jointly with K. Rama studied systematically the relations between the categories of (a) Compact Complex spaces, (b) Moishezon spaces, (c) algebraic spaces, (d) Kaehler spaces and constructed three classes of compact complex manifolds M which are Moishezon but not Kaehler. The non-Kaehlereity of M is obtained in these examples respectively basing on the ideas (a) for Kaehler M necessarily its even betti numbers are positive (b) Blanchard's necessary condition for a fibration : $X \xrightarrow{F} M$ with M, F Kaehler, is not satisfied. (c) on Kaehler M an algebraic 1-cycle cannot be algebraically equivalent to zero. These manifolds become Moishezon by demanding them to satisfy 'Siu's data'. In this note author constructs another class of compact complex manifolds, which are Moishezon but not Kaehler These are obtained from Thurston manifolds after suitable modification and these become non-Kaehler from the fact that for the modified Thurston manifolds M, $b_1(M)$ is odd whereas for a Kaehler M, necessarily odd betti numbers must be even and these are Moishezon via Siu's data again.

134 Hypersurfaces of A Space From With Commuting Curvature Tensor and Second Fundamental Form—*Sharief Deshmukh and S.I. Husain (Aligarh Muslim University, Aligarh).*

We consider the hypersurfaces of a space form $M(c)$ (Riemannian manifolds of constant curvature), satisfying $R(X, Y)\circ A = A\circ R(X, Y)$, where $R(X, Y)$ is curvature endomorphism which acts as derivation on Tensor Algebra and A is second fundamental form of the hypersurface. All hypersurfaces of this type have been classified. In particular, we prove

THEOREM : Let M be a hypersurface of space form $\overline{M}(c)$ satisfying

$$R(X, Y)\circ A = A\circ R(X, Y).$$

Then M is one of the following :

(*i*) flat

(*ii*) space form

(*iii*) locally Riemannian product $M^p(2c)XM^{n-p}(2c)$, in case $c < 0$, $n-p=p$.

(*iv*) locally Riemannian product $M^p(ka)XM^{(n-p)}(-kb)$ where $k=\sqrt{\beta^2+4c}$ and a, b are roots of $x^2-\beta x-c=0$, β being a real number.

135. Indefinite K-Space of Constant Holomorphic Sectional Curvature—*R.K. Nagaich and S.I. Husain (Aligarh Muslim University, Aligarh).*

In the present paper we prove an algebraic result on the curvature tensor of an indefinite K-space. This result has been already proved for Kaehler manifolds and Indefinite Kaehler manifolds (Ogiue, Nagaich and Husain) We also extend the result of Tanno on almost Hermitian manifolds to indefinite K-space. As an application we prove the results on axiom of holomorphic planes on indefinite K-spaces : The main results can be stated as follows :

THEOREM 1. Let (M, g, J) be an indefinite K space of dim ≥ 6. Then M has pointwise constant holomorphic sectional curvature if and only if $P(X, Y, X, JX) = 0$, for every orthonormal triplet $\{X, Y, JX\}$.

THEOREM 2. Let (M, g, J) be an indefinite K space of dim ≥ 4. Then M is of constant holomorphic sectional curvature if and only if $P(X, JX)X$ is proportional to JX for every unit tangent vector X.

136 **Bochner Flat Kaehler Manifolds With Commuting Curvature and Ricci Endomorphism** —*Viqar Azam Khan (Aligarh Muslim University, Aligarh)*

We start with a Bochner Kaehler manifold *i.e.* a Kaehler manifolds with vanishing Bochner curvature tensor, satisfying $R(x, y) \circ Q = Q \circ R(x, y)$, where Q is Ricci operator and classify all such manifolds. In particular we prove the following classification Theorem.

THEOREM : Let (M, J, g) be a connected Kaehler manifold with dimension $2n$ with vanishing Bochner curvature tensor and satisfying the condition

$$R(x, y) \circ Q = Q \circ R(x, y)$$

where R is the curvature tensor of M and Q is the Ricci tensor. Then M is one of the following :

(1) Flat,

(2) a complex space form,

(3) locally Riemannian product of two space forms of constant holomorphic sectional curvature c and $-c$ respectively.

137. **Totally Umbilical Submanifolds of A Complex Space Form**—*Shahid Ali (Aligarh Muslim Uuiversity, Aligarh).*

Using the classification Theorem for extrinsic spheres in Kaehler manifolds by Yamaguch. Nemoto and Kawabata, we classify all totally umbilical submanifolds of a complex space form $\bar{M}(c)$. Our main result is the following :

THEOREM : Let M be an n-dimensional $(n > 2)$ complete, simply connected totally umbilically submanifolds of a $2m$-dimensional complex space form $\bar{M}(c)$. Then M is one of the following :

(*i*) A complex space form $\bar{M}(c)$,

(*ii*) Totally real submanifold of constant curvature,

(*iii*) Isometric to an ordinary sphere,

(*iv*) Homothetic to a sasakian manifold.

(G) TOPOLOGY

138. **Common Fixed Points of a Pair of Mappings in Non-Archimedean Menger Spaces**—*S. L. Singh, (Gurukul Kangri University, Hariawar), and B. D. Pant, (Govt College, Gopeshwar, Chamoli).*

The main result of this paper is a common fixed point theorem for a quasi-contraction pair of weakly commuting mapping on a nonarchimedean Menger space. Our result improves a number of fixed point theorems in metric and probabilistic metric spaces.

139. Symmetry Group of Material Elements—*P. K Lal, (Bhagalpur University, Bhogalpur) and Subhtosh Bagchi (Murarka College, Bhagalpur).*

The branch of mechanics that treats materials as if they were continuous is called continuum mechanics. The foundations of continuum mechanics are based upon "smooth manifolds."

A manifold is a topological space which locally looks cartesian n-space ; it is built up of pieces of R^n glued together by homemorphisms. If these homeomorphisms are differentiable we obtain a differentiable manifold.

The original theory of simple materials was introduced by Walter Noll.

A "Material Element" is a septuple $(T, G, \pi, \Sigma, \hat{G}, \hat{S}, \hat{\gamma})$, where (T, G, π) is a body element, Σ is the "State space".

The paper proposes to study the material automorphism and hence introduce the concept of symmetry group of material elements.

140. Tolerance Visual Field and its Fuzziness—*S P. Bhagat (Bhagalpur University, Bhagalpur).*

The visual field X of an eye inherits a metric topology from the sphere concentric with the eye ball defining a tolerance relation τ as well as a fuzzy tolerance relation on it. A visual tolerance (also fuzzy tolerance) field and a visual tolerance Lobe can be connected by a relation which establishes set (fuzzy set) tolerance isomorphic structures.

141. F_ω Can not Support a Group Structure - *A K. Desai (Gujarat University, Ahmedabad).*

A countable homogeneous space which is Frechet but not metrizable was constructed by S P. Franklin and M Rajagopalan [1]. They denoted this space by F_ω. A question was raised there whether the space F_ω can support a group structure. We answer the question negatively.

142. Coincidence Theorems, Fixed Point Theorems and Convergence of the Sequences of Coincidence Values—*S L Singh (Gurukul Kangri University, Hardwar).*

Coincides theorems generalizing the coincidence theorems of Goebel and Park are proved for a pair of maps on an arbitrary set having values in a metric space. Apart from giving a few applications on normed spaces, some known fixed point theorems for a pair of commuting maps on a metric space are improved. Finally, new kind of convergence theorems for a pair of sequences of maps on an arbitrary set having values in a metric space are proved.

143. Characterizations of S-Compact Spaces—*Kavita Srivastava (Kulbhasker Ashram Degree College, Allahabad).*

The concept of S-compact spaces, an analogue of compact spaces, in which semiopen sets are used in place of open sets, was introduced independently by Dorsett [Bull Malaysian Math Soc. 4 (1981)] and by Prasad and Yadav [Indian J Math 24 (1982)]. In this paper we obtain some characterizations of S compact spaces.

144. On Negation of Topological Property—*S. Babusundar* (*University of Cochin, Cochin*).

Total negation of various topological properties are defined and examples given. Total negation of separation properties are studied in detail. Relations between total negation of topological properties and other properties are established and counter-example given.

The notion of anti-topological property is shown to be different from the concept of total negation of a topological property. A comparative study of these two concepts for many topological properties is made.

145. On Hyperconnected Spaces and Ultraconnected Spaces—*P. M. Mathew* (*University of Cochin, Cochin*).

A subset A of a topological space is said to be semi-open if there exists an open set U such that $U \subset A \subset \bar{U}$, where $\bar{U}$ denotes the closure of U. S. G. Crossley and S. K. Hildebrand used the concept of a semi-open set to define the notion of semi-homeomorphism. Properties preserved by semi-homeomorphisms are called semi-topological. In this paper we show that the property of being hyper-connected is semi-topological. We also establish that a hyperconnected door space is maximal hyperconnected and minimal door. Some order induced topological properties related to the concepts of hyperconnected and ultra-connected are discussed and it is shown that an ultra-connected $T_{1/2}$ spaces is both maximal ultra-connected and minimal $U_{1/2}$.

146. Cech Closure Operators and Reflexive Relations—*P.T. Ramachandran* (*University of Cochin, Cochin*).

Susan J. Andima and W.J. Thron investigated the pre-order $\leqslant$ associated with a topology T on X such that $a \leqslant b$ if and only if every open set in (X, T) containing b contains a. This is used to define order induced topological properties and to characterize the maximal topologies with respect to order induced topological properties. A dual characterization holds for minimal topologies only with the imposition of a further condition. In this paper we conduct an analogous study for Cech closure operators. The reflexive relation associated with a Cech clousure operators. The reflexive relation associated with a Cech closre operator is investigated. It is then used to definea closure property induced by a reflexive relation property and to give characterizations for maximal and minimal closure operators with respect to closure properties induced by reflexive relation properties. Here no further condition is necessary in the case of minimal Cech closure operators.

147. Super Connected Spaces—*P.M. Mathew and P.T. Ramachandran* (*University of Cochin, Cochin*).

A topological spacies called hyper-connected if there does not exist any disjoint pair of open sets. It is called ultra-connected if there does not exist any disjoint pair of closed sets. It is super-connected if it is both hyper-connected and ultra-connected. Many properties of super connected spaces are studied. Connections with other topological properties are studied. Connections with other topological properties are mentioned. The lattice of open sets of a super connected space is also investigated.

148. A Note on Mesocompact Spaces—*D. K. Thakkar* (*Saurashtra University, Rajkot*).

In this note we prove that (1) Closed, K-quotient image of every mesocompact space is mesocompact ; (2) Closed, presequential image of every sequentially mesocompact space is sequentially mesocompact ; and (3) Quasi-perfect image of a mesocompact space is mesocompact.

149. On Wallman Type Order Compactification—*Sunder Lal* (*Meerut University, Meerut*).

Choe and Park [Pacific J. Math. 82 (1979)] constructed Wallman type order compactification of a convex topological space equipped with a semi-closed order. In the case of discrete order this construction reduces to the Wallman compactification of a T_1 space. The main purpose of this note is to introduce an alternate, but simpler approach for the Wallman type compactification of an ordered space. This is accomplished by observing the topological nature of a convex semi-closed ordered space. We also prove two theorems as partial generalization of results by Singal and Lal [Proc. Amer. Math. Soc 83 (1981)].

150. A Generalization of Semi-Continuous Functions—*A. R. Singal and D. S. Yadav* (*Meerut University, Meerut*).

Using the concept of a semi-open set due to Levine (Amer. Math. Monthly, 1963), we introduce the notion of a slightly semi-continuous function, which is a simultaneous generalization of semi-continuous functions of Levine (*ibid*) and slightly continuous functions of Jain (Thesis, Meerut University, 1981). We also study preservation properties of this new class of functions.

(H) MEASURE THEORY, PROBABILITY THEORY, STOCHASTIC PROCESSES AND INFORMATION THEORY

151. A Production Process with General Probability Distributions—*A. Krishnamoorthy and M.J. Jacob* (*University of Cochin, Cochin*).

We consider a system which produces an item at random time points and the demands for the item also occur at random time points. Time between production and the inter-arrival times of demands are i i d. random variables with general distributions. We compute $p_k(t)$, the probability that there are k units in the system at time t and $W_n(t)$, the probability distribution of the waiting time of the n^{th} arrival for the following cases.

(*a*) When demands are for one unit and production is one unit at a time.

(*b*) When demands occur for one unit and production is 'a' (> 0) units at a time.

(*c*) When a random number of units is demanded and a random number of units is produced each time.

The expression for virtual waiting time is obtained for case (*a*). Also the case with vacation for the production process is considered.

152. General Arrival, Bulk Service Queues with Vacations to the Server—*M.J. Jacob and A. Krishnamoorthy* (*University of Cochin, Cochin*).

We consider a bulk service queueing model in which after the service of a batch, if the queue size is less than a fixed number '*a*', the server takes rest for a random length of time. Inter-arrival times are assumed to be *i.i.d.* random variables with a general distribution. Service is done in bulk where the maximum group size is *b* and minimum is *a*. Service period is exponentially distributed with the parameter depending on the group size. We find, using matrix geometric method, the steady state probabilities for the system size. The waiting time distribution for an arriving unit in the queue is also obtained.

153. Multi Server Queue with Restricted Birth and Death to Servers—*M. J. Jacob and A. Krishnamoorty*, (*University of Cochin, Cochin*).

We consider a multi-server queue in which after each service completion there is a positive probability p for the server to go out of the system and probability $1-p$ to remain in the system. There is an independent arrival process of the servers to the system, which is assumed to be Poisson with rate λ_j, if j is the number of servers present in the system at the arrival point. We take $\lambda_m = 0$ so that the maximum number of channels possible is $m(\geqslant 1)$. The numoer of servers available at each instant is a r.v. Using matrix geometric methods the steady state probabilities for the system size are written in the case of Poisson arrivals and exponential services. The steady state conditions are obtained explicitly. Also the particular cases for $m=1$ and $m=2$ are considered.

154. The Uncorrelated Multivariate Normal Distribution which is Closest to a Given Multivariate Normal Distribution—*J. N. Kapur* (*Indian Institute of Technology, Kanpur*).

The uncorrelated multivariate normal distribution which is closest to a given correlated multivariate normal distribution depends in the measure of distance (or of directed divergence) used. Here we find this distribution by using measures due to Kullback—Leibler, Renyi, Havida, Charvat, Bhattacharya, Chernoff and Matisuta. The closeness of this distribution is compared with that of the distribution obtained from the marginal distributions of the given correlated multivariate normal distribution.

155. New Information Theoretic Mathematical Models in Finance and Marketing—*Uma Kumar* (*Carleton University, Ottawa, Canada*), *and J.N. Kapur* (*Indian Institute of Technology, Kanpur*).

New Mathematical models based on concepts of information theory are proposed for (i) loss of information due to aggregation of financial statements (ii) brand switching and interdependence in marketing. The results of Lev, Herniter and Theil are generalized by making use of new measures of information, the concept of useful information due to Belis and Guiasu and measures of interdependence due to Watanabe and Kapur.

156. Characterization of a Quantitative-Qualitative Measure of Inaccuracy —*H.C. Taneja and R.K. Tuteja.* (*M.D. University, Rohtak*).

A quantitative-qualitative measure of inaccuracy is suggested and is characterized under a set of assumptions. Some properties of the new measure are discussed and also an application of the measure in determining the weighted code-word length has been studied.

157. On the Additivity Property of Degree β-Useful Information - *Priti Jain,* (*Maharshi Dayanand University, Rohtak*).

The weighted additivity plays an important role in the study of 'useful' information. Many authors characterized measure of 'useful' information by assuming utility of the joint experiment as the product of the utilities of individual experiment. But this definition implies that the utilities of the joint events will not be monotone functions and it is very difficult to apply such a definition in real life. In this paper we consider utility of the joint event as the monotone functions of the number of joint experiments. By using this propety we characterize a measure of 'useful' information.

158. Objective Systems Analysis and Modeling OSAM - *M. Hemarao,* (*Indian Inst. of Computer Tech. Cochin*).

In recent years under the discipline of Artificial Intelligence and Pattern Recognition Theory, Perceptron type algorithms are developed for objective modeling of complex systems. These algorithms use number of heurisites and are popularised under objective Systems Analysis and Modeling (OSAM) and also with different names like Group Method of Data Handling (GMDH) technique. Each algorithm involves the following three steps :

(*a*) Choosing a class of models.

(*b*) Estimating the model parameters.

(*c*) Testing the validity of model by using a selection criterion.

The major advantages of OSAM are

(1) It is able to select optimal model.

(2) The trained data is used to learn the system

(3) The noise stability models can be obtained.

(4) The effective components are found out.

(5) It detects the abnormal behaviour of the system in advance.

The paper describes the OSAM in the area of technical cybernetics by referring its developments stage by stage. The functional diversity of different models and importance of multicriterion environment is explained. Finally the directions of future work are given.

(I) NUMERICAL ANALYSIS APPROXIMATION THEORY, AND COMPUTER SCIENCE

159 Application of pade approximants to solution of integral equations—*P. Achuthan and S. Sunder (IIT, Madras).*

We present here the results of our recent study on the solution of certain integral equations using the Pade approximant techniques. It is shown that Fredholm theory of integral equations with degenerate kernels enables us to express the solution of such equations in terms of infinite power series from which the diagonal Pade approximants are constructed. We discuss here mainly the application of the Pade approximant approach in Quantum Scattering theory. The approximations are worked out numerically to compute scattering phase shift for the S-wave taking into account two types of potentials : (i) Exponential and (ii) Yukava. The results are compared with those obtained by other methods. The efficiency of the application of the Pade approximantion techniques for solving different kinds of integral equations is also stressed.

160. Splitting of continued fractions and the pade table—*P. Achuthan and S. Ponnusamy (Indian Institute of Technology, Madras).*

Occasionally one needs to look at mathematical functions in split forms as for example, symmetric-antisymmetric periodic-nonperiodic and even- odd. In the case of continued fractions (CF) for a given function expressed as a power series, the even-odd splitting is particularly interesting in as much as it is defined by and relates itself to the Pade' approximants in the following sense. Of even part of the given CF, the regular convergents occupy the even positions of the ordinary convergents of the given CF. Of the odd part, the regular convergents come in the positions of odd convergents of the original CF. The main connection between Pade approximants and continued fractions is that the entries of the Pade table can be realized as convergents of suitably chosen CFs. This is illustrated here with some specific examples. Various types of expansions of CFs of given functions are considered. The patterns of convergents of CFs identified as elements of the Pade takle are also studied in detail.

161. A note on papers of M.M. Derriennic and B. Wood—*H.S. Kasana and P.N. Agrawal (University of Roorkee, Roorkee).*

Durrmeyer (These de 3e cycle, Faculte' des Sciences the l' 'Universite' de Paris, 1976) introduced the following integral modification of the Bernstein polynomial :

$$M_n\{f(t), x\}=(n+1)\sum_{\nu=1}^{n} p_{n,\nu}(x)\int_0^1 p_{n,\nu}(t)\,f(t)\,dt,$$

where

$$p_{n,\nu}(x)=\binom{n}{\nu}x^{\nu}(1-x)^{n-\nu},\ \nu=0, 1, 2, \ldots, n.$$

This operator is, of course, similar to the well-known Bernstein-Kantorovitch polynomial. Derriennic (J. Approx. Theory 31 (1981), 325-343) studied some approximation properties of these operators in the ordinary and simultaneous approximation.

Recently, Wood (J. Approx. Theory 41 (1984), 51-55) considered the linear combinations $L_n(f(t), k, x)$ due to May (Canad. J. Math. 28 (1976), 1224-1250) and Rathore [Ph. D. Thesis, I.I.T. Delhi, (1973)] of these operators and obtained the order of uniform approximation by $L_n\{f(t), k, x)\}$ in terms of higher order modulus of continuity of f.

The purpose of this paper is to sharpen and give simple proofs of some of Derriennic's results. Also, an alternate form of the combinations considered by Wood has been introduced which makes it possible to develop a simpler proof of his Lemma 2.2, a main tool used to prove the only theorem of his paper and the determination of polynomials $Q(2k+1, k, x)$ and $Q(2k+2, k, x)$ occurring in the Voronowskaja type asymptotic formula, the last result of this paper. This result also shows that the order of approximation by the operators M_n is improved by taking the linear combinations.

162. Approximation and Strong Approximation in Locally Convex Spaces—*S. Elumalai* (*University of Madras*).

P. L. Papini [(J. Approx. Theory (1978)] has discussed the notions of best approximation best coapproximation and strong approximation in normed linear spaces. The main aim of this paper is to show that these concepts can be as well discussed in locally convex spaces. Incidentally, a statement made by Papini is shown to be false and an assumption made by him to be superflous.

163. Exponential Spline Technique for One Dimentional Heat and Wave Equations—*D.N. Holla* (*Space Applications Centre, Ahmedabad*).

In this paper, we consider the derivation of general implicit finite difference approximations of one dimensional heat and wave equation. General finite difference representations of second order partial differential equations are obtained by approximating the space derivatives with proper interpolation parameter. Time derivatives are approximated either by forward or central difference for heat or wave equation as the case may be. Exponential spline function has good approximation properties when compared with ordinary cubic or quadratic spline function in certain cases as reported in the literature by Rentrop.

Derivation of the finite difference scheme is restricted to one dimentional space variable. The technique can be generalized to higher dimensional heat or wave equation by the methods described by the author in his earlier papers. Many of the well known schemes reported in the literature can be derived as a special case of the general scheme for the particular choice of the parameter. Stability and Truncational errors of the general schemes are derived and discussed for stability and accuracy. Schemes are tested with numerical examples and results are compared for their merits. Continuous approximate solution of the differential equation can be obtained by determining interpolating exponential spline function from known discrete solution of the differential equation.

164. Some Properties of Etol Array Systems—*Nalinakshi Nirmal, R. Rana and C. Sri Hari Nagore (MIT, Madras).*

N. Nirmal and K. Krithivasan have introduced the notion of ETOL Array Systems and discussed their properties [1]. The classical concept of finite index for ETOL Systems is investigated by G. Rozenberg and D Vermier [3]. The authors in [2] extended the definition of finite index to ETOL Array Systems and studied its properties. In this paper, we discuss certain interesting closure properties of ETOL Array Systems of finite index and study the inter relation between the subfamilies of ETOL Array Systems of finite index.

(J) OPERATIONS RESEARCH

165. Application of Functional Analysis to Find Out Minimum Time Control on Certain Product Space—*R. N. Mukherjee (University of Burdwan Burdwan).*

The author developed a uniform theory of time optimal control problem of linear systems by functional analytic approach. In functional analytic approach the control function is essentially required to belong to an appropriate Banach space, so the problem can be formulated as a mapping from this Banach space to another. In this paper the different group of components of the control vector can be viewed as an element of a suitable Banach space. Consequently control vector can be looked upon as an element of the cartesian product of the appropriate number of possible different Banach space. Here an example has been taken, where the constraint on one component of the control vector is an integral constraint while that on the other component is an amplitude constraint. The Reachable set has been drawn to find out the time optimal control and the minimum time. The problem is to drive the system from any given initial state to some desired state in minimum time under the constraint $\|u\| \leqslant 1$, where u is the optimal control and $\|\cdot\|$ denotes the norm in an appropriate Banach space.

166. An Inventory System With Two Servers and Rest Periods—*Jacob K. Daniel (Govt. Arts College, Krishnagiri), and R. Ramanarayan (University of Cochin, Cochin).*

This paper is an investigation of an inventory system with two servers S_1 and S_2 and rest periods to the servers. The servers S_1 has to order a large quantity than that S_2 has to order. We assume that the servers take rest alternately. All the distributions considered in this model are general. The transition probabilities and system size probabilities are obtained.

167. On A Double Channel Queueing System With Zero/One Step Memory Input—*S D. Sharma (Kurukshetra University, Kurukshetra).*

This paper studies the transient and steady state behaviour of a double channel queueing system through the probability generating functions in their Laplace transforms. The input to the system has (i) Zero step memory, (ii) one

step memory, (iii) the inter-transition time follows the exponential distribution. The steady state solution is derived from the transient state and finally, some particular cases are discussed.

168. Existence of Almost Optimal Solution to Certain Linear and Non-Linear Problems in Operation Research Through Fixed-Point Alogrithm—*Dinkar Prasad Mishra, Tej Onkar Singh and N.P.S. Bawa (Government Science College, Rewa, M.P.).*

In this joint paper we have tried to investigate the existence of Almost Optimal Solutions to some non-linear one by using the techniques of M.J. Todd, B.C. Eaves and of O. Merril through fixed point alogrithms. We have also tried to compare the relative optimality of solutions by the use of different techniques. We have also tried to show how these alogrithms can be applied in the field of Economics.

169. Reliability and Cost Analysis of A Non-Maintained Parallel Redundant System Under Over Loading Effect—*P.P. Gupta and R. K. Sharma (M.M. College, Modinagar).*

This paper deals with the reliability behaviour and cost function analysis of a non-maintained parallel redundant complex system incorporating the concept of over loading effect. Complex system consists of two sub-systems *A* and *B*, connected in series. Sub-system *A* consists of two units in parallel whereas *B* has only one unit. All the units suffer three types of failures, *viz.*, human failure due to critical human errors, hardware failures and self life failures. The failures times for the system follow exponential time distribution. Using Laplace Transform technique, the Laplace Transform of the probabilities being in various states have been computed, which have been inverted further so as to obtain time dependent probabilities. Moreover, an important parameter of reliability *viz.*, M.T.T.F. (Mean time to failure) has also been calculated in addition to reliability and cost function analysis of the system. A numerical example has also been discussed in the end to highlight the important results.

170. Performance Measure for A General Queueing Network Model—*Vinod Kumar (Carleton University, Ottawa, Canada), O. Hawaleshka (University of Manitoba, Winnipeg, Canada) and J.N. Kapur (Indian Institute of Technology, Kanpur).*

Expressions for Expected Production Function, Variance of Production Function, Mean Queue Lengths and System Interdependence have been obtained for a General Queuing Network Model of Flexible Manufacturing Systems. Results for the Closed Queuing and Semi-Open Queueing Network Models are obtained as special cases of these more general results.

(K) FLUID DYNAMICS

171. 'Non-Similar' Solutions of MDG-Boundary Layer Flow in the Presence of A Transverse Magnetic Field—*R.N. Jat and J.L. Bansal* (*University of Rajasthan, Jaipur*).

The present paper is concerned with the 'non-similar' solutions of the two-dimensional boundary layer flow of an electrically conducting gas, in the presence of a transverse magnetic field. Applying Illingworth-Stewartson transformation and modifying the similarity variable suitably, the solutions of the boundary layer equations are obtained for a class of 'non-similar' flows. The equations have been integrated numerically taking $Pr = 1$, $\gamma = 1{\cdot}4$ and for three values of Mach number viz. 0, 1 and 2 in two cases of wall conditions viz. (i) adiabatic wall and (ii) heated wall. The effects of magnetic field and Mach number on skin-friction and heat transfer rate are studied.

172. Similarity Solutions for Two-Dimensional Magnetogasdynamic Boundary Layer Flow in a Transverse Magnetic Field—*J. L. Bansal and R.N. Jat* (*University of Rajasthan, Jaipur*).

The conditions of the similarity solutions of a two-dimensional boundary layer flow of an electrically conducting gas, in the presence of the transverse magnetic field at small magnetic Reynolds number, are studied. It is found that, in general, the similarity requirements involve as many as eleven parameters out of which eight are the same as that of non-magnetic gas-dynamic boundary layer flow. An Illustrative solution is obtained for power law velocity distribution, in Illingworth variables, at the outer edge of the boundary layer and for the corresponding variable magnetic field. The Prandtal number of the fluid is taken as unity and the magnetic field effects are confined to the boundary layer only.

173. Wave Interaction on Water of Variable Depth—*N. Nirmala and M.J. Vedan* (*University of Cochin, Cochin*).

The interaction of waves on water of variable depth is studied using derivative expansion method. It is seen that the wave interaction is possible and there is exchange of energy between different wave numbers. The total energy is not conserved in this interaction.

174. Vortex Method for Surface and Interfacial Waves—*K.V. Pramod and M.J.Vedan* (*University of Cochin, Cochin*).

The waves at the interface of two inviscid, incompressible fluids are studied, taking into account the vorticity at the interface. The free surface wave motion is obtained as a limiting case.

175. Two-Dimensional Vortex Flow of an Incompressible, Inviscid Fluid—*George Mathew, M.J.Vedan* (*University of Cochin, Cochin*) *and B.V. Baby* (*Bharata Mata College, Cochin*).

The two-dimensional nonpotential flow of an incompressible, inviscid fluid is modelled using a $U(1)$ Gauge invariant Lagrangian. The corresponding energy momentum tensor is also developed.

176. MHD Visco-elastic Boundary Layer Flow past a Stretching Plate — *Subhas Abel* (*Gulbarga University, Gulbarga*).

This paper deals with the study of Viscoelastic bourdary layer flow past a stretching plate, under the influence of magnetic field H_0 perpendicular to the plate.

The effect of magnetic field on the flow is investigated.

177. Numerical Solution of the Flow of A Second-order Fluid through a Channel With Porous Walls Under a Transverse Magnetic Field—*H G. Sharma and K.R. Singh* (*University of Roorkee, Roorkee*)

The problem of steady flow of an electrically conducting incompressible non-Newtonian second-order fluid through a channel with porous walls has been solved by finite difference technique using the Newton Raphson method. The behaviour of the velocity profile for different values of second-order parameter, Hartmann and Reynolds numbers is shown graphically. The results are compared with those obtained by the regular perturbation technique.

178 A Note on Jet of Power-Law Fluid—*V.Sirohi* (*Computer Centre , I I.T. Madras*), *M.G. Timol and N.L. Kalthia* (*S.V. Regional College of Engineering and Technology, Surat*).

The two-dimensional jet flow of Newtonian fluid emerging from an orifice has been discussed by Schilichting, Na. Kapur, any so many other researchers. Among them Kapur has studied a similar problem for the power-law fluids. The main difficulty in solving any jet problem is the nature of homogeneous boundaries of the governing non-linear ordinary differential equations. One cannot apply any initial value method to solve such equations by any numerical integration scheme like Range-kutta, Predictor-corrector methods etc.

Recently Na has suggested a method to solve Newtonian jet problem. In the present paper we extend the method to solve jet flow of non-Newtonian power law fluid for all flow indices except $n = 1/2$. The comparison of the present results with those obtained by Kapur is also discussed.

179. ZOLD to Describe Fluctuations in Invertebrate Populations — *V.N. Sharma and N. Joshi* (*Govt. Degree College, Jaiharikhal, Garhwal*).

The basic principles of analyses of data on entomological collections are outlined. Preston's (1948) lognormal distribution pattern of species abundances fits universally. However, in this paper mention is made of the use of the zero order logarithmic distribution (*ZOLD*) function to describe fluctuations in invertebrate populations. Calculations using Kendeigh (1979) data show the importance of the *ZOLD* in predicting population size when prevailing conditions are known.

180. Analytic Functions in Lidar Studies—*V.N. Sharma* (*Govt Degree College, Jaiharikhal, Garhwal*) *and M.K. Gupta* (*Govt. College, Kamand, Tehri Garhwal*).

The purpose of this paper is to describe the various analytic functions known to be adopted in laser monitoring of the atmosphere. Emphasis is given to the importance of the function used in Prandtl mixing length theory to permit detailed mapping of atmospheric turbulence.

180. **On Disturbances in a Semi-infinite Piezoelectric Rod due to an Impact at One End.**—*H.S. Chakraborty (Kalyani University, Kalyani).*

The methods of transform calculus are utilised to obtain an appropriate expression of the distribution of temperature and displacement in a uniform semi-infinite piezoelectric medium due to an applied force at one end.

181. **Stability of Viscous Flow Between Contra-Rotating Coaxial Cylinders**—*A.K. Ganguli (Patna University, Patna).*

Linear stability theory has been used to determine the stability criteria for the flow of incompressible viscous fluid between contra rotating cylinders with large angular velocity ratio. It has been observed by Chandrasekhar (1961) that an a symptotic relation exists in the form $T_c(1-\mu)^4 \to \tau$) $a_c(1-\mu) \to q$ when $1-\mu$ is large where T_c and a_c are critical Taylor number and wave number respectively and μ is the ratio of angular velocities. In the present paper a characteristic value problem has been derived. Our differential system consists of a fourth order and a second order equation in terms of the disturbance velocities u and v besides the boundary conditions. Expanding $v = \sum_0^\infty C_m e^{-mx} \sin mx$, we solve the fourth order equation in u and subtitute in the second order equation. Multiplying by $\sin nx$ and integrating we arrive at a set of linear equations in the constants C_m. Elimination of these constants give rise to a secular infinite order determinant involving the parameters $T^* = T(1-\mu)^4$ and $\gamma = (1-\mu)a$. In the first approximation the critical values of T^* and γ are obtained as $\tau = T_c^* = 1067$ and $1/q = 1/c = \frac{1}{2}$. These results compare favourably with those obtained by Chandrasekhar.

182. **A Note on the General Solution of the Heat Equation in a Moving Solid**—*C.K. Sharma (A.P.S. University, Rewa).*

The purpose of this note is not only to summarize the general solutions of heat equation and not subjected to introduce slowly converging series expansions but also generalized the problem when solid is moving along one of its direction.

183. **Stokeslet in a Half Space With Slip at the Wall**—*Sunil Datta, (Lucknow University, Lucknow).*

Singularities are very important in the construction of solution of problems in fluid mechanics and in this paper we determine the flow field induced by the fundamental singularity stokeslet of the Stokes equations describing the flow of a viscous incompressible fluid at low Reynolds number, in the presence of a plane wall on which slip takes place. Both the cases when the Stokeslet is aligned parallel to the wall and when it is perpendicular to the wall have been discussed. The solution has been effected by the help of double Fourier transform. It has been found that in general the effect of the slip decreases as the Stokeslet is moved away from the wall.

184. Unsteady Thermal Boundary Layers on a Porous Plate in a Rotating System—*G.N. Purohit and B. Lal* (*University of Rajasthan, Jaipur*).

An exact solution of energy equation including viscous dissipation has been obtained for the flow of an incompressible viscous fluid past a porous plate with uniform suction, the temperature of the plate is assumed to vary with time. Both the plate and the fluid are in a state of solid body rotation with constant angular velocity about z-axis normal to the plate. For time $t \leqslant 0$, there exists a steady velocity and temperature distribution while for $t > 0$, the temrerature of the plate is changed to an assumed function of time. Assuming that the existing velocity distribution (for $t \leqslant 0$) remains unperturbed, the subsequent temperature distribution (for $t > 0$) has been obtained using the Laplace Transform technique. The temperature distribution for different particular ceses has been numerically worked out.

185. On Free Convection Flow of Water at 4°C Past on Infinite Porous Wall—*Krishna Lal* (*Banaras Hindu University, Varanasi*).

The boundary layer equations for the free convection flow of water at 4°C past an infinite vertical porous wall have been discussed for the case of constant suction at the wall. Expressions for the velecity and temperature distributions have been derived and tabulated values are given for the velocity and temperature components. Expressions of rates of volume flow parallel to the wall for velocity and temperature have been calculated. The rate of monentum flow for the velocity and thermal boundary layers have also been derived. It is concluded that both shearing stress and Nusselt number depend on G (the Grashof number) and E (the Eckert number).

186. Effect of Suction on Oscillatnry Flow Past an Infinite Vertical Porous Plate With Mass Transfer—*Ashok Kumar* (*Kurukshetra University, Kurukshetra*).

In this paper a two-dimensional unsteady flow of a viscous in-compressible fluid past an infinite vertical porous plate is studied under the following conditions :

(*i*) There is a variable suction.

(*ii*) The free stream velocity oscillates about a non-zero constant mean.

(*iii*) The plate is at a constant temperature.

(*iv*) The free convection currents are present near the plate.

(*v*) The foreign mass like H_2, H_e ; H_{2o} etc. are present.

The solutions of the coupled non-linear equations governing the flow are obtained approximately. The effect of suction parameter on species concentration at different distances are shown graphically. The effect of Grash of Number on mean skin friction is also shown graphically. The numerical values of mean rate of heat transfer, mean skin friction are tabulated. During the course of study the effects of various parameters like E (Eckert Number), γ (Suction parameter), G_r (Grashof Number) are discussed.

(L) ELECTROMAGNETIC THEORY, MAGNETO-HYDRODYNAMICS, ASTRONOMY AND ASTROPHYSICS

187. Vertical Motion of a Sphere in an Electrically Conducting, Stratified, Diffusive Medium—*S.P. Anjali Devi and M.R. Raghavachar* (*Bharathiar University, Coimbatore*).

The vertical (upwards) flow of an incompressible, Newtonian, viscous, stratified, diffusive, electrically conducting fluid past a non-conducting sphere, embedded with a uniform magnetic field applied in the vertical direction is considered. Quasi-steady approximation is made allowing for a time dependent buoyancy force. Method of matched asymptotic expansions is employed to obtain the drag exerted on the sphere, for small values of a stratification parameter α such that $\alpha \ll 1$, $R_e \ll \alpha^{1/3}$, $F_r^2 \ll \alpha^{-1/3}$, $P_e \gg \alpha^{2/3}$ and $R_m = o(\alpha^{2/3})$. The drag is computed for certain typical values of the magnetic interaction parameter and diffusivity parameter. The results reveal the fact that while the stratification and magnetic field have the enhancing effect over the drag, the effect of diffusivity is to decrease it.

188. Strong Cylindrical Hydromagnetic Shock in a Rotating Gas—*S Kumar* (*University of Maiduguri, Maiduguri, Nigeria, and R. Prakash* (*Agra College, Agra*).

Chisnell-Chester-Whitham method has been used to study the propagation of strong cylindrical diverging shock wave through a rotating gas in the presence of a magnetic field having all the three components. Solid body rotation and the presence of magnetic field make the medium non-uniform. Assuming an initial density distribution $\rho_0 = \rho' r^w$, were ρ' is the density at the axis of symmetry and ω is a constant, case of strong shock is explored under two conditions (*i*) when the ratio of densities on either side of the shock nearly equals $(\gamma+1)/(\gamma-1)$; where γ is the adiabatic index of the gas, is small or (*ii*) when the applied magnetic field is large.

The expressions for the pressure, the density and the radial particle velocity immediately behind the shock have been derived for both the cases.

(M) BIOMATHEMATICS

189. A Bifurcation Phenomenon in Two Species Models—*K. Kuppu Swamy Rao* (*Osmania University, Hyderabad*).

The classical two species model describing the dynamics of a predator interaction is given by

$$N_1' = N_1 (b_1 - a_{11}N_1 - a_{12}N_2) \; ; \; N_2' = N_2(-b_2 + a_{21}N_1 - a_{22}N_2)$$

where N_1 and N_2 denote respectively the prey and predator population sizes and b_1 and $-b_2$ their inherent net birth rates in the absence of any constraints. The coefficients a_{ij}, $i \neq j$, measure the effect of the interaction of the two species whereas a_{ii} denotes the self inhibition coefficient. If $a_{11} > 0$, the two equilibrium points in the right half plane are given by

$$E_1 = [(b_1a_{22} + b_2a_{12}) / D, (b_1a_{21} - b_2a_{11}) / D] ; E_2 = (b_1 / a_{11}, 0),$$

where $$D = a_{11}a_{22} + a_{12}a_{21}.$$

If $b_2 > b_1a_{21}/a_{11}$, there is no equilibrium in the first quadrant. If $b_2 < b_1a_{21}/a_{11}$. the equilibrium lies in the first quadrant. If $b_2 = b_1a_{21}/a_{11}$, E_1 and E_2 coincide. Finally, E_2 remains unchanged for all the values of b_2. Viewing b_2 as a parameter, E_1 is an equilibrium which bifurcates from stationary equilibrium. Similary in case of two competing species, the model equations are

$$N_1' = N_1(b_1 - a_{11}N_1 - a_{12}N_2) ; N_2' = N_2 b_2 - a_{21}N_1 - a_{22}N_2),$$

where b_i and a_{ij} are positive constants. The three non-trivial positions of equilibria are

$$E_1 = [(b_1a_{22} - b_2a_{12})/D, (b_2a_{11} - b_1a_{21})/D] ; E_2 = (b_1/a_{11}, 0) ;$$

$$E_3 = (0, b_2/a_{22}),$$

where $$D = a_{11}a_{22} - a_{12}a_{21}.$$

Considering the two cases $D > 0$ and $D < 0$ separately and treating the equilibrium as a function of the parameter b_2, E_1 can be seen as a branch of equilibria connecting the equilibrium E_2 with branch E_3. In the classical models the situations describe the growth and interaction of species with constant parameters in a constant environment. But a more realistic model must take into consideration periodic and seasonal effects of food supply influenced by hunting and harvesting cycles, weather conditions and mating habits. In this paper a study is attempted to investigate the bifurcation phenomenon under these constraints which may be reflected by taking the coefficients as periodic functions of time.

190. Varied are the Applications of Mathematics—*S. M. Uppal (Kenyatta University, P. O. Box 43844, Nairobi, Kenya)*

The paper discusses how Mathematical theory often appeared to be treated initially as a pursuit for its own sake but that subsequent treatment in various fields of application show how abstract theory can often be used to solve problems in our physical world. Examples of usage of Mathematics to solve problems in Physics are given, including use of Euclidean Geometry, Partial Differential Equations and group theory. More recent applications in the field of Biology are mentioned where computer technology has resulted in an acceleration of Mathematical processes to solve problems. The author deals with present progress and also problems in using mathematics in the Social Sciences, citing the difficulties of handling non quantitative and random effects and asking whether the Social Sciences may lead to new branches of Mathematics being discovered to deal with such difficulties. Reference is also made to the great use of Mathematics in Economics which has become, increasingly a mathematical discipline and also to the development in Mathematical linguistics where the basic structural properties of all languages can be described mathematically. In view of the current need for other branches of knowledge to call on recent mathematical discoveries, the author hints that this need will continually grow.

191. A Model for the Growth of Infection—*G. P. Pokhariyal* (*University of Nairobi, Nairobi, Kenya*).

In this paper a model for the growth of infection is constructed. By discussing the observed effects, it is said that the growth of infection mainly depends upon (1) structure of tissues, (2) the environment and (3) the progeny and their survival. A mathematical model for growth of infection that takes the form of negative exponential function is constructed and various measures are mentioned. The control strategy for the infection is also discussed.

(N) MISCELLANEOUS

192. Blood Flow in the Arterial System under Body Acceleration – *V.K. Sud*, (*All-India Institute of Medical Sciences, New Delhi*) *and G. S. Sekhon* (*Indian Institute of Technology, New Delhi*).

In the present work, the finite element technique is used to analyze the effect of body acceleration on blood flow through a model of the human arterial system. The human cardiovascular system is modelled after Avolio (1980) as an arrangement of fairly large number of interconnected arteries. The branches of the arterial tree are terminated at various points of the human arterial tree. The conditions at the terminal nodes are prescribed in terms of rate of out-flow. The fluid pressure at one of the nodes is prescribed or known a' priori. The compensatory action of the heart pump is also assumed to be given. Matrix equations governing the model are derived. A computational scheme is worked out. Results of computation under typical conditions of body acceleration are presented and discussed.

193. Initial Creation—Its Consequences–I—*S. C. P. Halakatti* (*Karnatak University, Dharwar*).

The incompleteness of the Big-Bang theory in providing the answers to the problems, (*i*) initial creation, (ii) the origin of the turbulence with regard to the origin of structure in the universe, (iii) the origin of space and time-has been pointed out. It has been contended that the urge for the better understanding of these problems leads us to probe into pre-Big-Bang situation, which will take us to many interesting consequences.

194. Solution of 15 Problems of Mathematics—*D.M. Patel* (*10 Pritama Society, Navrang Pura, Ahmedabad*).

The purpose of this paper is to complete the solutions of 15 problems which are enumerated on the pages 370—376 of volume I of "The World of Mathematics" by James R. Newman.

195. On Agricultural Eco-Systematics—*R.N. Lal and Kumkum Dutta (Bhagalpur University, Bhagalpur).*

Eco-system Ecology and evolution tied to systematics becomes eco-systematics, which has a wide application to agricultural research. The paper proposes to outline the role of an eco-systematics in : (*a*) *mapping* the agriculture ecosystems of a country giving good estimates of potential yield of usual and unusual crops in years with usual weather and unusual weather, (*b*) *making* croping recommendations based on analysis of local flora in remote areas ; (*c*) *predicting* the amount of the biomass and potential energy available in various agroeco-systems and also wherein in a state and elsewhere exotic crops and pests, even endangered Species are most likely to gain an ecological foot hold ; (*d*) *deducing* the climate of remote areas.

196. Stratified two-phase sampling results when there is non-response—*R.M. Sekkappan, University of Papua New Guinea).*

In this paper, we generalize the results of Sekkappan (1984) and Bahadur Singh and Sedransk (1978) by considering a more general mutiple analysis of covariance model with homoscedastic errors. Here we assume that the regression parameters are common in all strata and further the stratum sizes are unknown. It is shown that our estimators include as special cases all the estimators of Sekkappan (1984) and Bahadur Singh and Sedransk (1978) when there is non-response.

197. On Jyotirmimansa and Its Nearness to Modern Trends—*T. Thrivikraman (University of Cochin, Cochin).*

K.V. Sarma has published with an introduction, the sanskrit text Jyotirmimansa of Nilakanta. In the introduction he has pointed to the special nature of this one unlike other ancient texts on astronomy. In this paper, we bring out some of those special points which make the approach of the text very near to the modern approach.